Part I Problems and Solutions

Problem 1:

Find the solution satisfying the initial conditions:

$$y'' - y = x^2, \qquad y(0) = 0, \qquad y'(0) = -1$$

Solution: $y_h = c_1 e^x + c_2 e^{-x}$.

Try:

$$\begin{aligned} y_p &= a_1 x^2 + a_2 x + a_3 \\ y_p'' &= 2a_1 \\ x^2 &= -a_1 x^2 - a_2 x + 2a_1 - a_3 \end{aligned}$$

Thus $a_1 = -1, a_2 = 0, a_3 = -2$. So

$$y = c_1 e^x + c_2 e^{-x} - x^2 - 2$$

$y(0) = 0$ gives us $c_1 + c_2 - 2 = 0$; $y'(0) = -1$ gives us $c_1 - c_2 = -1$, so $c_1 = \frac{1}{2}, c_2 = \frac{3}{2}$. Thus,

$$y = \frac{1}{2}e^x + \frac{3}{2}e^{-x} - x^2 - 2$$

Problem 2: *Find a particular solution to the DE*

$$y'' - y' - 2y = 3x + 4$$

Solution: $y_p = Ax + B \to 0 - A - 2(Ax + B) = 3x + 4 \to A = -\frac{3}{2}$, $B = -\frac{5}{4}$, so $y_p = -\frac{1}{4}(6x + 5)$

Problem 3: *Find a particular solution to the DE*

$$y^{(3)} + 4y' = 3x - 1$$

Solution: Since $y' = 0$ for $y =$constant, try

$$\begin{aligned} y_p &= Ax^2 + Bx = x(Ax + B) \\ y_p' &= 2Ax + B \\ y_p'' &= 2A \\ y_p''' &= 0 \end{aligned}$$

Thus, $y_p''' + 4y_p' = 4(2Ax + B) = 3x - 1 \rightarrow 8A = 3, 4B = -1$, so $A = \frac{3}{8}$ and $B = -\frac{1}{4}$. Thus,

$$y_p = \frac{1}{8}\left(3x^2 - 2x\right)$$