Amplitude Response: Pole Diagram Applet

Open the applet and play with it. The main graph window is actually a 3D plot that you can rotate with your mouse.

Exploring the Applet

The applet shows the pole diagram and amplitude response of the system

$$\ddot{x} + b\dot{x} + kx = k\cos(\omega t).$$

It is designed to illustrate the connection between the pole diagram and the amplitude response of the system.

To return the graph window to its original position click on the 'top' button. The screen should look something like
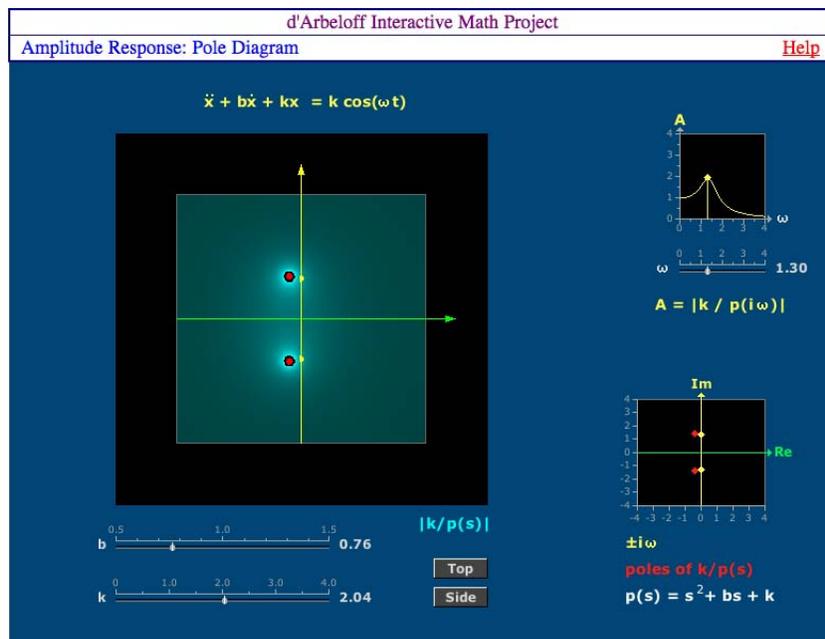
The main graph window and the pole diagram in the lower right look the same. Notice that the real axis on both is green, the imaginary axis is yellow, and the poles are red. The amplitude response graph in the upper right is the same one we've seen in the `Amplitude and Phase Second Order` applets.

The amplitude response graph is the same color as the imaginary axis because $A = \frac{k}{|p(i\omega)|}$, that is, because $i\omega$ is on the imaginary axis. The yellow

dot in all the windows indicates the current value of ω. (Because $\cos(\omega t)$ is the real part of $e^{i\omega}$ and $e^{-i\omega}$, there are dots at both $\pm i\omega$ in the pole diagram and main graph.)

Play with the b, k and ω sliders to see how the poles and amplitude change. Now set $b = 0.75$ and $k = 2$, then move ω to the position where the system has the maximum amplitude response.

Now click on the 'side' button and rotate the main graph so you can see all its features. It should look something like this:
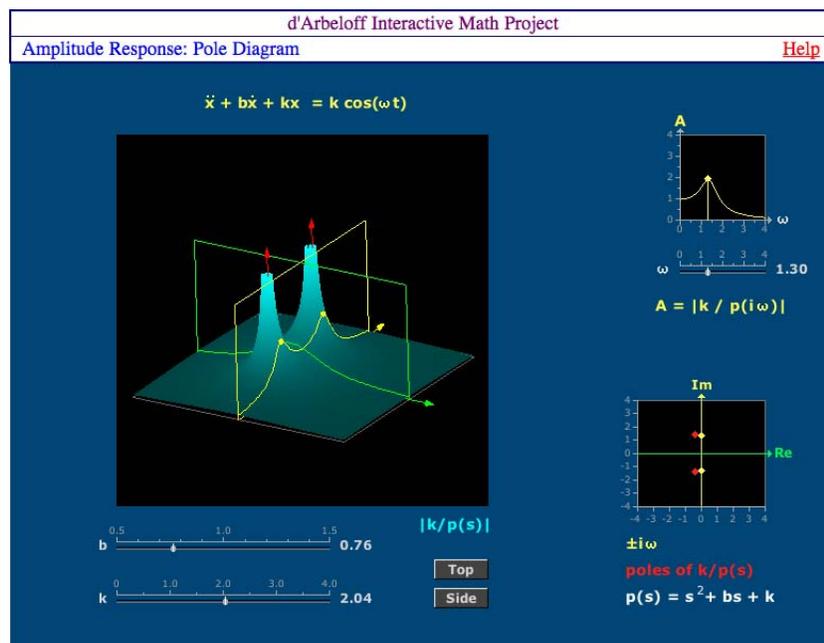
The surface shown in the plot is the graph of the magnitude of the transfer function $\left|\frac{k}{p(s)}\right|$. The yellow curve on the surface above the imaginary axis is therefore the plot of $\left|\frac{k}{p(i\omega)}\right|$. Notice that this is the same graph as the amplitude response graph in the upper right of the applet. (The main graph also shows $\left|\frac{k}{p(i\omega)}\right|$ for $\omega < 0$. This is just the mirror image of the graph for $\omega > 0$.

We can now explain how the main graph illustrates why choosing $i\omega$ near a pole gives a big response: The yellow amplitude curve runs alongside the "mountains" that rise up near the poles. As $i\omega$ gets close to a pole, the amplitude curve moves up the side of the mountain.

Questions

1. Why can't the amplitude response become infinite by placing $i\omega$ directly on a pole?

2. Why are the poles in the left half-plane ($\text{Re}(s) < 0$) for all choices of b and k?

3. Move b to 1.5 and k to 0.4. What happens to the poles? At what frequency is the amplitude response maximized?

4. Leave k at 0.4 and move b to 1.1. The poles are now a complex conjugate pair. What frequency gives the maximum amplitude response?

Answers

1. Because $i\omega$ must stay on the imaginary axis, and the poles are *not* on the imaginary axis.

2. Because both b and k are positive, we know that the system is stable (since all second order systems with positive coefficients are stable). Therefore, all the poles must have negative real part, which places them in the left half-plane

3. At these settings of b and k the poles are real. The peak amplitude response occurs at $i\omega = 0$.

4. The peak amplitude is still at $\omega = 0$. As we saw in the session on Frequency Response in unit 2, this system needs to be sufficiently lightly damped, not just underdamped, in order to have a practical resonant frequency.

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