Part II Problems and Solutions

Problem 1: *[Step and delta] For each of the following functions $f(t)$, (i) draw a graph, (ii) draw a graph of the generalized derivative, (iii) write a formula for $f(t)$ and for $f'(t)$ (with possibly a few values not defined) using $u(t-a)$, $\delta(t-a)$, and other functions.*

(a) $f(t) = 0$ *for* $t < 0$, $f(t) = -t$ *for* $t > 0$.

(b) $f(t) = 0$ *for* $t < 0$, $f(t) = 1 - t$ *for* $t > 0$.

(c) $f(t) = 0$ *for* $t < 0$, $f(t) = 2t - 1$ *for* $0 < t < 1$, $f(t) = 0$ *for* $t > 1$.

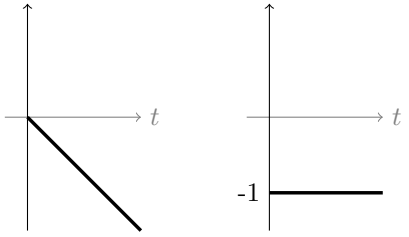
(d) $f(t) = 0$ *for* $t < 0$, $f(t) = t - \lfloor t \rfloor$ *for* $t > 0$, *where* $\lfloor t \rfloor$ *denotes the greatest integer less than or equal to* t.

Solution: **(a)** $f(t) = -u(t)t$, $f'(t) = -u(t)$.

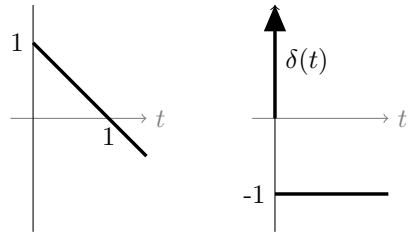
(b) $f(t) = u(t)(1-t)$, $f'(t) = -u(t) + \delta(t)$.

(c) $f(t) = (u(t) - u(t-1))(2t-1)$, $f'(t) = 2(u(t) - u(t-1)) - \delta(t) - \delta(t-1)$.

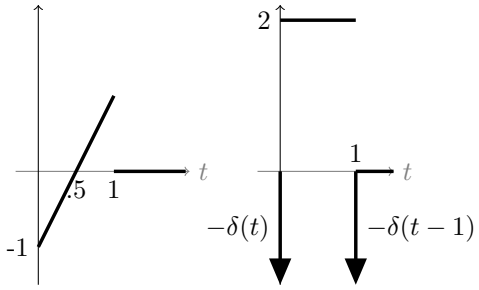
(d) $f(t) = (u(t) - u(t-1))t + (u(t-1) - u(t-2))(t-1) + (u(t-1) - u(t-2))(t-2) + \cdots$
$= u(t)t - u(t-1) - u(t-2) - \cdots$.
$f'(t) = u(t) - \delta(t-1) - \delta(t-2) - \delta(t-3) \cdots$.



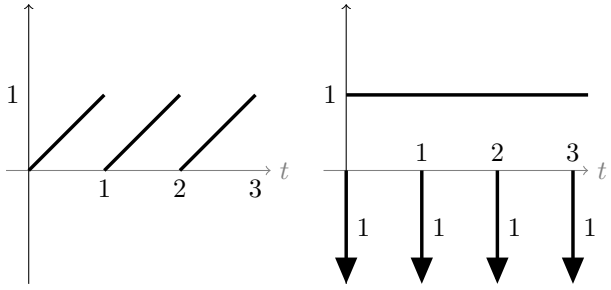
(1a) $f(t)$ and $f'(t)$



(1b) $f(t)$ and $f'(t)$



(1c) $f(t)$ and $f'(t)$



(1d) $f(t)$ and $f'(t)$

Note: In the graphs for (1d) we used that convention that the weight 1 next to the down arrow indicates $-\delta(t)$, $-\delta(t-1)$ etc.

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