Matrix Notation

Exercise. The system (which we looked at earlier)

$$\dot{x} = x + 3y$$
$$\dot{y} = x - y$$

has general solution

$$x = 3c_1e^{2t} - c_2e^{-2t}$$
$$y = c_1e^{2t} + c_2e^{-2t}.$$

Re-express this using matrix notation. What are two independent basic solutions?

Answer. The matrix form for the system is

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

and the solution can be expressed as

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3c_1e^{2t} - c_2e^{-2t} \\ c_1e^{2t} + c_2e^{-2t} \end{pmatrix} = c_1e^{2t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} + c_2e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Two basic independent particular solutions are

$$e^{2t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

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