PROFESSOR: Welcome in this recitation. So we're going to talk about linear systems of equations. So in the first question, we are given a system of equations. x dot equals 6x plus 5y. y dot equals x plus 2y. We're asked to write this system in matrix form. The second part asks us to convert a differential equation of second order, x dot dot plus 8x dot plus 7x equals to zero into matrix form, basically into system of ODEs, similar to the first part. In the third part of the problem, we're asked to interpret the population model x dot equals 2x minus 3y, y dot equals x minus y.

So here, x and y are modeling either a prey or predator. And you're asked to think about the interpretation of the system to determine which of x or y is the prey or the predator. So why don't you take a few minutes? Think about these three questions, and I'll be right back.

Welcome back. So for the first question, basically, we're asked to write this system in matrix form. So we have [x, y] derivative for that left-hand side. You need to write this in the form of a matrix multiplying x and y. So here, we would have 6, 5; 1, 2. And that would be our system of differential equations in matrix form. And what we would be solving for would be the vector [x, y].

The second part of the problem, we need to do the opposite, go from the second order differential equation into matrix form. So to do that, we introduced a new variable, y equals x dot. And from that point, we can then write x dot dot-- so if I'm going to just start with what we know about the equation, x dot dot equals-- let me write it in a system first before I do it in a vector form. We would write x dot dot equals minus 7x minus 8x dot. But we introduced a new variable x dot equals 2y. So we have minus 7x minus 8y.

So now, the other equation we need is the one that tells us what this y is. So we have x dot equals to y, which is the new variable that we introduced here. And so we go from a second-order differential equation into a system of two differential equations that we can write now in vectorial form, in matrix form, like we did for the first part, as x, x dot which is just y-- I'm just going to write this like this, it's just from what we defined-- equals to, again, [x, y], like we did previously. And now we have to read off our system to find the coefficient of this matrix.

So x dot equals to y means that there is zero coefficient in front of the x, a 1 in front of the y. x dot dot equals minus 7x. So we will have a minus 7 multiplying the x and minus 8 multiplying the y. And so that's how we convert a differential equation, second order, into the systems of differential equations in matrix form. And this matrix would be called, referred to, the companion matrix of this differential equation. OK, so that ends the second part.

So now for the third question, we're asked to interpret this population dynamics system of equation. Minus 3y; y dot equals x minus y. So the question was, we have two species. Which one is the prey, which one is the predator? So how do we go about figuring this out? Let's look at how x dot varies with y or basically variable x varies with y.

Here, we can see that we have a coefficient that is minus 3. It is negative, which means that when y increases, we have a more and more negative x dot, which means that the value of x goes down. So as the population y increases, we have a decrease of population x, which suggests that y is a predator eating up population x.

And if you look at the equation for y, we have x minus y. And here, what we see is that when x increases, the population y then increases. So that definitely confirms that y is our predator that basically increases by feeding on the population x. And as it feeds on population x, y increases, which means that here this term becomes more and more negative, which means x decreases in turn. And these two terms could be modeling, for example, here just the growth of the population, so birth term of the prey. And these minus y here could be just modeling, for example, a death rate of these predators. And so we have x prey and y predator.

So from this recitation, we learned how to convert a system of differential equations to matrix form. We learned how to convert a second-order differential equation into also matrix form, or basically system differential equation, introducing notion of companion matrix. And we learned how to interpret a system of differential equations in terms of what populations could it be modeling or what dynamics it could be modeling. So that ends the recitation.