18.04 Recitation 7
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1. Compute the Laurent series for $f(z) = \frac{z+1}{z^3(z^2+1)}$ on the region $A : 0 < |z| < 1$ centered at $z = 0$.

Ans: See Example 7.22 in the notes.

2. Find the Laurent series around $z = 0$ for $f(z) = \frac{1}{z(z-1)}$ in each of the following regions:

2.1. The region $A_1 : 0 < |z| < 1$

2.2. The region $A_2 : 1 < |z| < \infty$.

Ans: See Example 7.23 in the notes.

3. Suppose $f(z)$ is an analytic function on the unit disk such that $f(w) = 5$ for all $w \in \{x + i0 : -0.2 \leq x \leq 0.2\}$. What is $f(i/2)$? Does your answer change if instead the assumption is that $f(w) = 5$ for all $w \in \{x - 3i/4 : -0.2 \leq x \leq 0.2\}$?

Ans: Consider the function $g(z) = f(z) - 5$, which is analytic on the unit disk. Since $g(z)$ has a non-isolated zero at the point 0, it follows that g must be identically equal to 0. In particular, $0 = g(i/2) = f(i/2) - 5$, so that $f(i/2) = 5$.

4. Find a power series solution to the differential equation $f'(x) = f(x) + 2$ with $f(0) = 0$.

Ans: See Example 7.24 in the notes.

18.04 Complex Variables with Applications

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