18.04 Midterm 1 Review Session
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1. Basic complex arithmetic
1.1. Find real and imaginary parts of a complex number.
1.2. Express a complex number in polar form.
1.3. Euler's formula; doing trignometry using the exponential function; hyperbolic sine and cosine.
1.4. Finding n^{th} roots of a complex number.

2. More complex arithmetic
2.1. $\arg(z)$ and its branches; principal branch of $\arg(z)$; continuity of $\arg(z)$.
2.2. $\log(z)$ and its branches.
2.3. Complex powers; computing the power coming from the principal branch of $\log(z)$.

3. Complex functions as mappings
3.1. Behavior of horizontal/vertical/radial lines or circles under complex mappings.
3.2. Behavior of regions of the complex plane under complex mappings.

4. Analytic functions
4.1. Definition of complex derivative; understand directly from the definition of the complex derivative why $\overline{z}$ is not analytic.
4.2. Understand difference between continuity and differentiability.
4.3. Cauchy-Riemann equations; use CR equations to show that a function is analytic at some point; use CR to show that a function is not analytic at some point.
4.4. For $f = u + iv$ analytic, express f' only in terms of u (and its partials) or v (and its partials).
4.5. Region of analyticity for compositions of functions.

5. Line integrals
5.1. Compute line integrals explicitly.
5.2. Fundamental theorem of complex line integrals; using the fundamental theorem even in certain situations where the integrand is not analytic.
5.3. Path independence and Cauchy's theorem; simple connectedness.
5.4. Using Cauchy's theorem even in certain situations where f is not analytic on a simply connected region by splitting up the contour.
5.5. Extended Cauchy's theorem; reducing certain integrals over more general contours to integrals over circles.

6. Cauchy's integral formula (for derivatives)
6.1. Evaluating integrals using Cauchy's integral formula (for derivatives); isolating singularities by splitting the contour.
6.2. Computing real integrals using Cauchy's integral formula (for derivatives); triangle inequality for integrals.

7. More applications of Cauchy's formula
7.1. Analyticity of complex derivatives.
7.2. Cauchy's inequality.
7.3. Liouville's theorem.
7.4 Mean value property.
7.5. Maximum modulus principle; finding the minimum modulus in certain situations.