18.04 Practice Laplace transform, Spring 2018

On the final exam you will be given a copy of the Laplace table posted with these problems.

Problem 1.
Do each of the following directly from the definition of Laplace transform as an integral.

(a) Compute the Laplace transform of $f_1(t) = e^{at}$.

(b) Compute the Laplace transform of $f_2(t) = t$.

(c) Let $F(s) = \mathcal{L}(f; s)$. Prove the s-derivative rule: $\mathcal{L}(tf(t); s) = -F'(s)$.

Problem 2.
For each of the following you can use the Laplace table if it helps.

(a) Compute the Laplace transform of $\cosh(at)$.

(b) Compute the Laplace transform of $f(t) = \begin{cases} 0 & \text{for } t < 5 \\ \cosh(a(t-5)) & \text{for } t > 5 \end{cases}$.

(c) Compute the Laplace transform of $f(t) = \begin{cases} \sin(t) & \text{for } 0 \le t \le \pi \\ 0 & \text{for } t > \pi \end{cases}$

(d) Compute the Laplace transform of $t\cos(at)$.

(e) Let $\Gamma(z) = \mathcal{L}(t^{z-1}; s = 1)$. Show that $\Gamma(z+1) = z\Gamma(z)$

Problem 3.
(a) Use the Laplace transform to solve the differential equation $x' + x = te^{2t}$, with $x(0) = 3$.

Find the Laplace inverse using the formula involving the sums of residues. (Be sure to verify that the hypotheses of the theorem hold.)

(b) Solve $y' - y = \begin{cases} 0 & \text{for } t < 1 \\ 1 & \text{for } t > 1 \end{cases}$, with $y(0) = 0$.

Why does the inversion formula involving sums of residues not apply?

Problem 4.
Use the Laplace transform to solve the differential equation $x'' + x = \sin(t)$, with $x(0) = 0$, $x'(0) = 0$. (Hint: use the table to do the Laplace inverse.)