

Exam 1 Practice Questions II, 18.05, Spring 2014

Note: This is a set of practice problems for exam 1. The actual exam will be much shorter.

1. A full house in poker is a hand where three cards share one rank and two cards share another rank. How many ways are there to get a full-house? What is the probability of getting a full-house?

2. 20 politicians are having a tea party, 6 Democrats and 14 Republicans. To prepare, they need to choose:

3 people to set the table, 2 people to boil the water, 6 people to make the scones.

Each person can only do 1 task. (Note that this doesn't add up to 20. The rest of the people don't help.)

(a) In how many different ways can they choose which people perform these tasks?

(b) Suppose that the Democrats all hate tea. If they only give tea to 10 of the 20 people, what is the probability that they only give tea to Republicans?

(c) If they only give tea to 10 of the 20 people, what is the probability that they give tea to 9 Republicans and 1 Democrat?

3. Let C and D be two events with $P(C) = 0.25$, $P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$?

4. More cards! Suppose you want to divide a 52 card deck into four hands with 13 cards each. What is the probability that each hand has a king?

5. Corrupted by their power, the judges running the popular game show *America's Next Top Mathematician* have been taking bribes from many of the contestants. Each episode, a given contestant is either allowed to stay on the show or is kicked off.

If the contestant has been bribing the judges she will be allowed to stay with probability 1. If the contestant has not been bribing the judges, she will be allowed to stay with probability $1/3$.

Suppose that $1/4$ of the contestants have been bribing the judges. The same contestants bribe the judges in both rounds, i.e., if a contestant bribes them in the first round, she bribes them in the second round too (and vice versa).

(a) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

(b) If you pick a random contestant, what is the probability that she is allowed to stay during both of the first two episodes?

(c) If you pick random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

6. There is a screening test for prostate cancer that looks at the level of PSA (prostate-specific antigen) in the blood. There are a number of reasons besides prostate cancer that a man can have elevated PSA levels. In addition, many types of prostate cancer develop so slowly that that they are never a problem. Unfortunately there is currently no test

to distinguish the different types and using the test is controversial because it is hard to quantify the accuracy rates and the harm done by false positives.

For this problem we'll call a positive test a true positive if it catches a dangerous type of prostate cancer. We'll assume the following numbers:

Rate of prostate cancer among men over 50 = 0.0005

True positive rate for the test = 0.9

False positive rate for the test = 0.01

Let T be the event a man has a positive test and let D be the event a man has a dangerous type of the disease. Find $P(D|T)$ and $P(D|T^c)$.

7. Suppose now that events A, B and C are *mutually independent* with

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(C) = 0.5.$$

Compute the following: (Hint: Use a Venn diagram)

(i) $P(A \cap B \cap C^c)$ (ii) $P(A \cap B^c \cap C)$ (iii) $P(A^c \cap B \cap C)$

8. Suppose X is a random variable with $E(X) = 5$ and $\text{Var}(X) = 2$. What is $E(X^2)$?

9. Suppose I play a gambling game where I either win or lose k dollars. Suppose further that the chance of winning is $p = .5$.

I employ the following strategy to try to guarantee that I win some money.

I bet \$1; if I lose, I double my bet to \$2, if I lose I double my bet again. I continue until I win. Eventually I'm sure to win a bet and net \$1 (run through the first few rounds and you'll see why this is the net).

If this really worked casinos would be out of business. Our goal in this problem is to understand the flaw in the strategy.

(a) Let X be the amount of money bet on the last game (the one I win). X takes values 1, 2, 4, 8, \dots . Determine the probability mass function for X . That is, find $p(2^k)$, where k is in $\{0, 1, 2, \dots\}$.

(b) Compute $E(X)$.

(c) Use your answer in part (b) to explain why the strategy is a bad one.

10. (a) Suppose that X has probability density function $f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. Compute the cdf, $F_X(x)$.

(b) If $Y = X^2$, compute the pdf and cdf of Y .

11. Let X be a discrete random variable with pmf p given by:

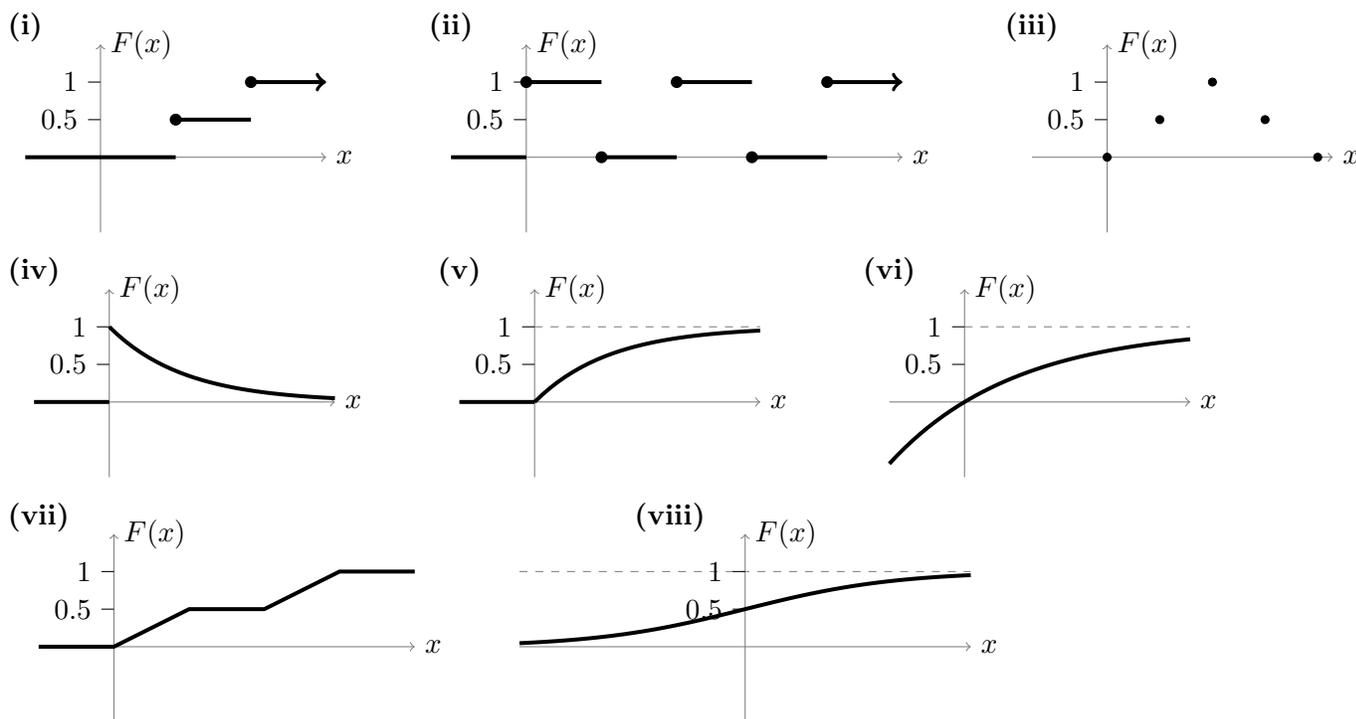
x	-2	-1	0	1	2
$p(x)$	1/15	2/15	3/15	4/15	5/15

(a) Let $Y = X^2$. Find the pmf of Y .

(b) Find the value the cdf of X at $-1/2, 3/4, 7/8, 1, 1.5, 5$.

(c) Find the value the cdf of Y at $-1/2, 3/4, 7/8, 1, 1.5, 5$.

12. For each of the following say whether it can be the graph of a cdf. If it can be, say whether the variable is discrete or continuous.



13. **Normal Distribution:** Throughout these problems, let ϕ and Φ be the pdf and cdf, respectively, of the standard normal distribution. Suppose Z is a standard normal random variable and let $X = 3Z + 1$.

- Express $P(X \leq x)$ in terms of Φ .
- Differentiate the expression from (a) with respect to x to get the pdf of X , $f(x)$. Remember that $\Phi'(z) = \phi(z)$ and don't forget the chain rule.
- Find $P(-1 \leq X \leq 1)$.
- Recall that the probability that Z is within one standard deviation of its mean is approximately 68%. What is the probability that X is within one standard deviation of its mean?

14. Compute the median for the exponential distribution with parameter λ . (The exponential distribution is defined on page 62 of the text.)

15. Let X and Y be two random variables and let r , s , t , and u be real numbers.

- Show that $\text{Cov}(X + s, Y + u) = \text{Cov}(X, Y)$.
- Show that $\text{Cov}(rX, tY) = rt\text{Cov}(X, Y)$.
- Show that $\text{Cov}(rX + s, tY + u) = rt\text{Cov}(X, Y)$.

16. (**Another Arithmetic Puzzle**) Let X and Y be two independent Bernoulli(.5) random variables. Define S and T by:

$$S = X + Y \quad \text{and} \quad T = X - Y.$$

- (a) Find the joint and marginal pmf's for S and T .
- (b) Are S and T independent.

17. Toss a fair coin 3 times. Let X = the number of heads on the first toss, Y the total number of heads on the last two tosses, and F the number of heads on the first two tosses.

- (a) Give the joint probability table for X and Y . Compute $\text{Cov}(X, Y)$.
- (b) Give the joint probability table for X and F . Compute $\text{Cov}(X, F)$.

18. (More Central Limit Theorem)

The average IQ in a population is 100 with standard deviation 15 (by definition, IQ is normalized so this is the case). What is the probability that a randomly selected group of 100 people has an average IQ above 115?

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