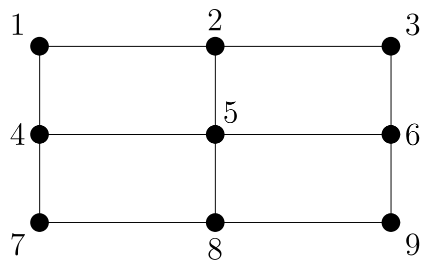
18.085 Computational Science and Engineering I
Fall 2008

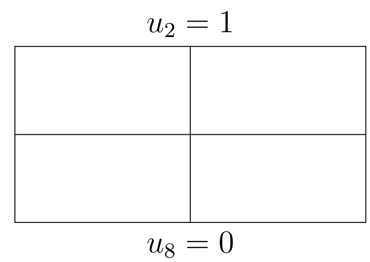
18.085 Quiz 2 November 3, 2006 Professor Strang

Your PRINTED name is: ____________________

Grading **1** **2** **3** **4**

1) **(25 pts.)** This network (square grid) has 12 edges and 9 nodes.





(a) Do not write the incidence matrix! Do not give me a MATLAB code! Just tell me:

(1) How many independent columns in A?

(2) How many independent solutions to $A^{\mathrm{T}}y = 0$?

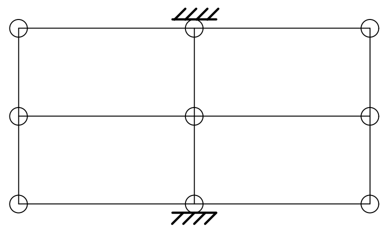
(3) What is row 5 (coming from node 5) of $A^{\mathrm{T}}A$?

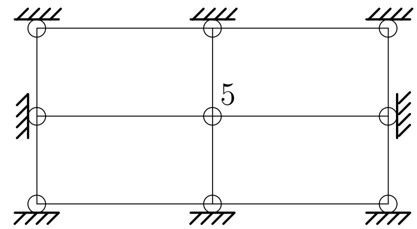
I do want the whole of row 5.

(b) Suppose the node 2 has voltage $u_2 = 1$, and node 8 has voltage $u_8 = 0$ (ground). All edges have the same conductance c. **On the second picture write** all of the other voltages u_1 to u_9. Check equation 5 of $A^{\mathrm{T}}Au = 0$ (at the middle node).

x

2) **(25 pts.)** Suppose that square grid becomes a plane truss (usual pin joints at the 9 nodes). Nodes 2 and 8 now have supports so $u_2^{\mathrm{H}} = u_2^{\mathrm{V}} = u_8^{\mathrm{H}} = u_8^{\mathrm{V}} = 0$.





(a) Think about the strain-displacement matrix A. Are there any mechanisms that solve $Au = 0$? If there are, **tell me carefully how many** and **draw a complete set**.

(b) Suppose now that all 8 of the outside nodes are fixed. Only node 5 is free to move. There are forces f_5^{H} and f_5^{V} on that node. The bars connected to it (North East South West) have constants c_N c_E c_S c_W. What is the (**reduced**) matrix A for this truss on the right? What is the **reduced** matrix $A^{\mathrm{T}}CA$? What are the **displacements** u_5^{H} and u_5^{V}? For 1 point, is that truss (fixed at 8 nodes) statically determinate or indeterminate?

xx

3) **(25 pts.)** This question is about the velocity field $v(x, y) = (0, x) = w(x, y)$.

(a) Check that $\operatorname{div} w = 0$ and find a stream function $s(x, y)$. **Draw** the streamlines in the xy plane and show some velocity vectors.

(b) Is this shear flow a gradient field ($v = \operatorname{grad} u$) or is there rotation? If you believe u exists, find it. If you believe there is rotation, explain how this is possible with the streamlines you drew in part (a).

xxx

4) **(25 pts.)** Suppose I use linear finite elements (hat functions $\phi(x) =$ trial functions $V(x)$). The equation has $c(x) = 1 + x$ and a point load:

$$\textsf{\textbf{Fixed-free}} \qquad -\frac{d}{dx}\left((1+x)\frac{du}{dx}\right) = \delta\left(x - \frac{1}{2}\right) \qquad \begin{array}{l}\text{with } \ u(0) = 0 \\ \text{and } \ u'(1) = 0\,.\end{array}$$

Take $h = 1/3$ with two hats and a half-hat as in the notes.

(a) On the middle interval from 1/3 to 2/3, $U(x)$ goes **linearly** from U_1 to U_2. Compute

$$\int_{1/3}^{2/3} c(x)\big(U'(x)\big)^2\, dx \quad \text{and} \quad \int_{1/3}^{2/3} \delta\left(x - \frac{1}{2}\right) U(x)\, dx\,.$$

Write those answers as

$$\begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} ? \\ ? \end{bmatrix}.$$

You have found the 2 by 2 "element stiffness matrix" and the 2 by 1 "element load vector."

(b) On the first and third intervals, similar integrations give

$$\begin{bmatrix} U_1 \end{bmatrix} \begin{bmatrix} ?? \end{bmatrix} \begin{bmatrix} U_1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} U_1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix};$$

$$\begin{bmatrix} U_2 & U_3 \end{bmatrix} \begin{bmatrix} 5.5 & -5.5 \\ -5.5 & 5.5 \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} U_2 & U_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Assuming your calculations and mine are correct, what would be the overall finite element equation $KU = F$? (Not to solve) xxxx