

18.100A Fall 2012: Assignment 8

Directions: Don't consult solutions to previous semesters' assignments ; if you collaborate, write up the solutions independently and list collaborators.

Reading: Chaps. 9, 10

Chapter 9 should be review of things you have had before; you can read it rapidly, noting what things are in it, so that you can go back to those you find you need in later work.

Chapter 10 requires reading, to establish the notation. The utility of the distinction made toward the end between “pointwise”, “local”, and “global” properties of functions will become clearer as the semester provides examples.

Prob. 1 (3)

a) Work 9.2/2

b) Work 9.3/3

c) The “Dirichlet function” $f(x)$ is defined to be 1 if x is rational, and 0 otherwise.

Prove that any rational $c > 0$ is a period of $f(x)$; this shows $f(x)$ is periodic, but has no minimal period.

Prob. 2 (3)

a) Work 10.1/1 (to use $f(I)$, where $f(x)$ is thought of as a map $I \rightarrow \mathbf{R}$).

b) Work 10.1/6b (needs 18.01 calculus)

c) Work 10.1/7b (this needs absolute values)

In these last two problems, the difficulty is mainly in getting the work in logical order, and then writing it up clearly. Do the work on scratch paper first.

Prob. 3 (2) Work 10.3/2

Do not use an indirect argument – argue directly. Focus on the conclusion: i.e., start with an arbitrary x_0 . What is it you wish to prove about $f(x_0)$?

(Use absolute value to express boundedness efficiently, as in Section 10.1, (2).)

Prob. 4 (2) Work 10.3/4 (numbered 5 in some very old printings)

For practice with the “for $x \approx a$ ” terminology.

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