18.100B : Fall 2010 : Section R2
Homework 4

Due Tuesday, October 5, 1pm

Reading: Tue Sept.28 : connected sets, convergence, Rudin 2.45-47, 3.1-7
Thu Sept.30 : no reading assingment due to **Quiz 1** (covering Rudin 1.1-38, 2.1-44)

1 . Consider the notes titled *Compactness vs. Sequentially Compactness* posted on the web page. Prove Lemma 3 stated on those notes.

2 . Assume (X, d) is a connected metric space. Prove that the only subsets that are both open and closed are X and $\varnothing$.

3 . If in a metric space (X, d) we have $B \subset A \subset X$, then the set B is a connected subset of (A, d) (i.e. A with the relative topology) if and only if B is connected subset of (X, d).

4 . Let $(a_n)_{n=1}^{\infty}$ be a sequence in $\mathbb{R}$ with the property that *no subsequence* converges. Prove that $|a_n| \to \infty$. Does the same property hold if the a_n are in $\mathbb{Q}$ and we consider $(a_n)_{n=1}^{\infty}$ as a sequence *in the metric space* $\mathbb{Q}$?