# 18.100C Lecture 17 Summary

Weierstrass' example of a nowhere differentiable continuous function.

**Theorem 17.1.** *Let $(f_n)$ be a sequence of functions $[a, b] \to \mathbb{R}$ which are everywhere differentiable. Suppose that $(f_n')$ is uniformly convergent. Suppose also that $(f_n(x_0))$ is convergent for just one point $x_0 \in [a, b]$. Then $(f_n)$ itself is uniformly convergent, the limit $f$ is differentiable everywhere, and*

$$f' = \lim_{n\to\infty} f_n'.$$

**Corollary 17.2.** *If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has convergence radius $\rho > 0$, then it is everywhere differentiable in $(-\rho, \rho)$, and its derivative is $f'(x) = \sum_{n=1}^{\infty} a_n n x^{n-1}$ (which has the same convergence radius).*

**Corollary 17.3.** *If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has convergence radius $\rho > 0$, then it is infinitely often differentiable in $(-\rho, \rho)$.*

**Theorem 17.4.** *Let $(f_n)$ be a sequence of functions $[a, b] \to \mathbb{R}$ which are everywhere differentiable. Suppose that there are constants $C, D$ such that $|f_n(x)| \leq C$, $|f_n'(x)| \leq D$ for all $n$ and $x$. Then $(f_n)$ has a uniformly convergent subsequence.*

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