18.311: Principles of Applied Mathematics — Rodolfo Rosales

Lecture 34 — Spring 2014

Introduce scalar product definition of self-adjoint, and use to prove:

- Eigenvalues of self-adjoint matrices are real.
- Eigenvectors of self-adjoint matrices corresponding to distinct eigenvalues are orthogonal.
- NxN self-adjoint matrices have N orthogonal eigenvectors. Use descending technique: reduce size of problem by one by showing the matrix keeps the hyperplane orthogonal to an eigenvector invariant. Then use:
- Any matrix has, at least, one eigenvalue. Follows from "any polynomial has, at least, one root."
- Scalar product not unique. Examples:
    - $\langle u, v \rangle = \text{Sum}_n\, w_n\, u_n^*\, v_n$, for some weights $w_n > 0$. One application: cost functions in optimization.
- Any self-adjoint matrix with positive eigenvalues gives rise to a scalar product: $u^* A v$. Prove this.