Describe general method, for a problem of the form $a(x, y)*u_x + b(x, y)*u_y = c(x, y)$, u given along some curve.
That is: $u = U(z)$ on some curve $x = X(z)$ and $y = Y(z)$.

Equations: $dx/ds = a$, $dy/ds = b$, and $du/ds = c$, to be solved with the conditions $x = X(z)$, $y = Y(z)$ and $u = U(z)$ for $s = 0$.
Leads to solution expressed in the form $u = u(s, z)$, with $x = x(s, z)$ and $y = y(s, z)$.
where:

- z = parameter/label for the characteristic curves.
- s = parameter that results from solving o.d.e.'s above = parameter along each characteristic curve.

(s, z) is a curvilinear, coordinate system. Characteristic coordinates.
To get the solution must solve for s and z as functions of x and y.

Illustrate the coordinates (s, z) graphically.

Another example: $x^2*u_x + x*y*u_y = y$, with conditions on circle $u = U(\zeta)$ for $x = \cos(\zeta)$, $y = \sin(\zeta)$.

$dx/ds = x^2$ and $dy/ds = x*y$ and $du/ds = y$
$x = \cos(\zeta)$ $y = \sin(\zeta)$ $u = U(\zeta)$ for $s=0$.

$x = \cos(\zeta)/(1-s*\cos(\zeta)) = r*\cos(\zeta)$; $r = 1/(1-s*\cos(\zeta))$;
$y = \sin(\zeta)/(1-s*\cos(\zeta)) = r*\cos(\zeta)$;
$u = - \tan(\zeta)*\ln(1-s*\cos(\zeta)) + U(\zeta)$;

In all the examples, the characteristic curves are independent of the solution. This follows from the equations being LINEAR. That is a, b, and c do NOT depend on u.

Next we move up to NONLINEAR PROBLEMS
Example 4: General kinematic wave equation $u_t + c(u)*u_x = 0$, with $u(x, 0) = f(x)$.

1. Characteristic form and characteristic speed.
2. Solution generally cannot be written explicitly.
3. Geometrical interpretation of the solution. Leads to a clear picture of how conservation is achieved: "SLIDING SLABS" image.
4. Wave distortion/steepening and wave breaking.
5. Smooth solutions do not exist for all time.
6. Show that the characteristics can/do cross in space-time.
7. Derivatives become infinity at time of first crossing.

NOTE: $dc/du < 0$ for traffic flow, and $dc/du > 0$ for river flows.
Consequences for wave steepening: waves steepen backwards (TF) or forwards (RF). Matches observations.