

PaK
12/12/05

More on Young tableaux

$$\textcircled{1} \text{ RSK } S_n \rightarrow \{(A, B) \mid A, B \in \text{SYT}(\lambda), \lambda \vdash n\}$$

cor $n! = \sum_{\lambda \vdash n} |\text{SYT}(\lambda)|^2$

New bijection: jeu-de-taquin (Aunt Cathy's sliding #

Schützenberger's bij $S_n \leftrightarrow \{(A, B)\}$ tile game)
 φ : take $\sigma \in S_n$, write from bottom left to top right as long diagonal, then slide everything up as much as can legally (+ still satisfy SYT ineq) then left as much as possible, then fill in corner hole by... um... example unclear
Actual bijection is α sending σ to $(\varphi(\sigma), \varphi(\sigma^{-1}))$

Theorem: α is a bijection between $S_n +$ what we want to be to be

Now let's go back to our earlier fascinating result

$$|\text{SYT}(\lambda)| = \frac{n!}{\prod_{x \in \lambda} h_x}$$

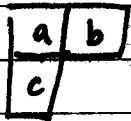
$$90s: |\text{SYT}(\lambda)| \cdot \prod h_x = n!$$

Let's try to make a bijection

$$S_n \leftrightarrow (A, \{f_x, x \in \lambda\}) \text{ where } A \in \text{SYT}(\lambda) \text{ and } f_x \text{ is some number to be explained later}$$

Reminder: ~~bubble~~ bubble sort

Now he's ready to define 2-d bubble sorting



Put numbers in λ willy-nilly. Bubble in order bottom \rightarrow top column by column, right to left, bubble w.r.t. b if $a > c > b$, c if $a > b > c$

Claim: 2-dim bubble sort is an equinumerous surjection $S_n \rightarrow \text{SYT}(\lambda)$, i.e. surjection $\forall A, A' \in \text{SYT}(\lambda) \quad |\text{pre-im } A| = |\text{pre-im } A'|$

For $\boxed{\quad \quad \quad \quad \quad}$, let $a_i = \# \text{ steps } \theta(i)$ makes to the right during bubble. This gives bij between permutations + inv. index ("obvious")

Analogously, for each position, start w/ all 0, if entry in that position's right neighbor swaps w/ it in A , then in this labelling add 1. If entry swaps w/ one below, swap labelling + subtract 1 from it

So each box has range of size its hook length, going from -height to +length

