18.330 Problem Set 10

Due in class: Fri 23 Apr 04

(28) Find the **LU** decomposition of the Pascal matrix $\underline{P}$ = $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{bmatrix}$

and then use it to ascertain the smallest eigenvalue of this matrix via repeated application — clearly explained and exemplified — of the so-called **inverse power method**. Also track down the largest eigenvalue of $\underline{P}$ via the normal power method ... and thus rediscover a very remarkable relationship between those two values.

(29) By iteration of one sort or another, solve (and describe how you solved) the appended two sets of equations from a textbook by Froberg:

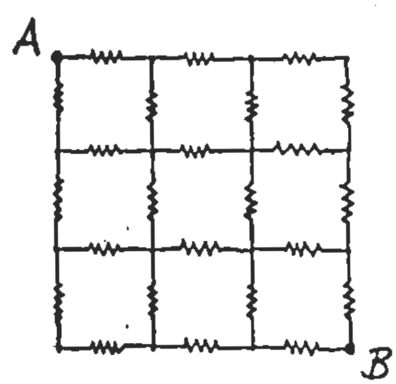
(a)

$$\begin{aligned} x_1 + 10x_2 + x_3 &= 10 \\ 2x_1 + 20x_3 + x_4 &= 10 \\ 3x_2 + 30x_5 + 3x_6 &= 0 \\ 10x_1 + x_2 - x_6 &= 5 \\ 2x_4 - 2x_5 + 20x_6 &= 5 \\ x_3 + 10x_4 - x_5 &= 0 \end{aligned}$$

(b)

$$\begin{aligned} x - 0.1\,y^2 + 0.05z^2 &= 0.7 \\ y + 0.3x^2 - 0.1\;\;xz &= 0.5 \\ z + 0.4y^2 + 0.1\;\;xy &= 1.2 \end{aligned}$$

(30) To dwell on an increasingly **sparse** matrix, consider the passive electrical circuit that consists of $2N(N-1)$ perfect one-ohm resistors wired into a square lattice such as pictured on the right for $N = 4$.



For $N = 2$ it is easy to calculate that the total resistance between diagonally opposite corners like A and B is precisely 1 ohm. For $N = 3$ it is $3/2$ ohm , and for $N = 4$ it is $13/7$ ohm .

What is that resistance when $N = 10$? HINT: Introduce voltages at all N^2 nodes, and prescribe (say) $V_A = +1$ and $V_B = -1$ volt .