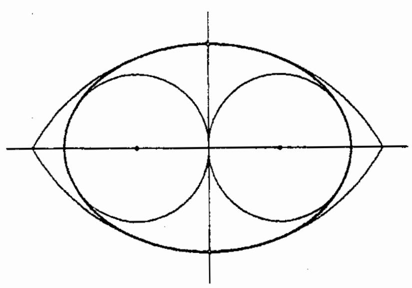
reused as <u>18.330 Handout</u> Wed 03 Mar 04

<u>**18.023 Handout**</u> Tue 01 Oct 96

Circumference of the ELLIPSE ... and of the 4-leaf CLOVER

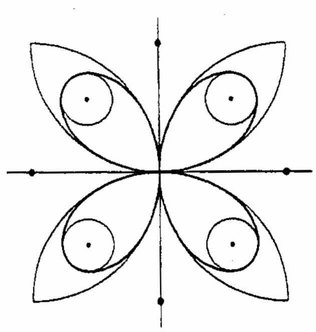
MORAL OF THIS STORY: Even the humble MIDPOINT (or trapezoidal) RULE for estimating a definite integral sometimes converges amazingly fast — especially when the integrand happens to be smooth and periodic, and the range of integration covers exactly one period ... or 2 or 3, etc.



# evals		x 4	error
1	1.923824745243	7.695298980971	.054903402916
2	1.910225893795	7.640903575181	507997126
3	1.910100855256	7.640403421023	7842967
4	1.910098931335	7.640395725340	147285
5	1.910098895281	7.640395581125	3070
6	1.910098894531	7.640395578124	68
7	1.910098894514	7.640395578057	2
<u>8</u>	1.910098894514	<u>7.640395578055</u>	0
9	1.910098894514	7.640395578055	0
10	1.910098894514	7.640395578055	0

Ellipse $x^2/2 + y^2 = 1$

$$S \;=\; 4\int_{\alpha=0}^{\pi/2} \sqrt{1+\sin^2\alpha}\, d\alpha \;\simeq\; 7.640396$$



# evals		x 16	error
1	.620911766612	9.934588265796	.246140045248
2	.606071354194	9.697141667105	8693446557
3	.605559659691	9.688954555054	506334507
4	.605530252947	9.688484047153	35826606
5	.605528189697	9.688451035150	2814602
6	.605528028536	9.688448456572	236025
7	.605528015078	9.688448241241	20694
8	.605528013901	9.688448222422	1874
9	.605528013795	9.688448220722	174
10	.605528013785	9.688448220564	16
11	.605528013784	9.688448220549	2
<u>12</u>	.605528013784	<u>9.688448220548</u>	0
13	.605528013784	9.688448220548	0
14	.605528013784	9.688448220548	0

Clover $r = \sin 2\theta$

$$S \;=\; 8\int_{\theta=0}^{\pi/4} 2\sqrt{\cos^6\theta + \sin^6\theta}\, d\theta \;\simeq\; 9.688448$$