18.443. Pset 2. Due Wednesday, Sep 20.

(1) (Similar to example in lecture 2.) Take a height of men data subset (second column in body_men.mat file on the class website). Use 'dfitool' to fit normal and log-normal distributions (or, if necessary, gamma). If you think that a fit looks good (or not) perform a chi-squared test to see if the test agrees with you.

(2) Find MLE of a parameter λ for Poisson distribution $\Pi(\lambda)$. Compute Fisher information and state asymptotic normality property of MLE.

(3) Prove that if $X_1, \ldots, X_n \sim U[0, \theta]$ then MLE $\hat{\theta} = \max(X_1, \ldots, X_n)$ is consistent, i.e. $\hat{\theta} \to \theta$ in probability. Find a constant c such that $c\hat{\theta}$ is unbiased estimate of θ.

(4) Consider a parametric family of distributions with the p.d.f. given by

$$f(x|\theta) = \begin{cases} e^{\theta - x}, & \text{when } x \geq \theta, \\ 0, & \text{when } x < \theta, \end{cases}$$

and where $-\infty < \theta < \infty$. Find MLE $\hat{\theta}$.

(6) For multivariate parameter $\theta = (\theta_1, \ldots, \theta_k)$ prove that Fisher information matrix

$$I(\theta) := \mathrm{Cov}(\nabla l(X|\theta)) = -\mathrm{E}\mathrm{Hess}(l(X|\theta)).$$