18.906: Problem Set IV

Homework is an important part of this class. I hope you gain from the struggle. Collaboration can be effective, but be sure that you grapple with each problem on your own as well. If you do work with others, you must indicate with whom on your solution sheet.

Extra credit for finding mistakes and telling me about them early!

16. **(a)** Show that the canonical bundle over $\mathrm{Gr}_k(\mathbb{R}^n)$ is indeed locally trival.

(b) Express the tangent bundle of $\mathrm{Gr}_k(\mathbb{R}^n)$ in terms of the canonical bundle $\xi_{n,k}$ and its orthogonal complement $\xi_{n,k}^\perp$.

17. Let $p : P \to B$ be a principal G-bundle.

(a) Construct a natural trivialization of the pull-back $p^*P = P \times_B P \downarrow P$.

(b) Let F be a left G-space. Let $\overline{P}$ denote the left G-space with underlying space P and G-action given in terms of the right action on P by $g \cdot x = xg^{-1}$. Construct a natural function from the set of continuous equivariant maps $\overline{P} \to F$ to the set of continuous sections of $P \times_G F \downarrow B$. (Hint: The fiber bundle $P \times_G \overline{P} \downarrow B$ admits a canonical section.) Then construct a natural map the other way. (Hint: What is the pullback of $P \downarrow B$ along $P \times_G F \to B$?) Show that these two functions are inverses.

18. **(a)** Construct a principal action of the Lie group $SU(2)$ on S^7. What is the orbit space?

(b) Let M be a compact smooth manifold (which perforce admits the structure of a CW complex) of dimension at most 4. Show that

$$\mathrm{Bun}_{SU(2)}(M) \cong H^4(M; \mathbb{Z}) \, .$$

19. Verify Prop. 57.2 in the notes.

20. $\varnothing$

21. $\varnothing$

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