Open Problem 3.1 *Does there exists a constant $c > 0$ such that it is NP-hard to, given ϕ, and G distinguis between the cases*

1. $h_G \leq \phi$, *and*
2. $h_G \geq c\sqrt{\phi}$*?*

It turns out that this is a consequence [RST12] of an important conjecture in Theoretical Computer Science (see [BS14] for a nice description of it). This conjecture is known [RS10] to imply the Unique-Games Conjecture [Kho10], that we will discuss in future lectures.

Conjecture 3.10 (Small-Set Expansion Hypothesis [RS10]) *For every $\epsilon > 0$ there exists $\delta > 0$ such that it is NP-hard to distinguish between the cases*

1. *There exists a subset $S \subset V$ with $\mathrm{vol}(S) = \delta\,\mathrm{vol}(V)$ such that $\frac{\mathrm{cut}(S)}{\mathrm{vol}(S)} \leq \epsilon$,*
2. $\frac{\mathrm{cut}(S)}{\mathrm{vol}(S)} \geq 1 - \epsilon$, *for every $S \subset V$ satisfying $\mathrm{vol}(S) \leq \delta\,\mathrm{vol}(V)$.*

References

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Fall 2015