Open Problem 8.1 *Is the Unique Games conjecture true? In particular, can it be refuted by a constant degree Sum-of-squares relaxation?*

Remarkably, approximating `Max-Cut` with an approximation ratio better than α_{GW} is has hard as refuting the Unique Games Conjecture (UG-hard) [KKMO05]. More generality, if the Unique Games Conjecture is true, the semidefinite programming approach described above produces optimal approximation ratios for a large class of problems [Rag08].

Not depending on the Unique Games Conjecture, there is a NP-hardness of approximation of $\frac{16}{17}$ for `Max-Cut` [Has02].

Conjecture 8.2 *For any $\epsilon > 0$, the problem of distinguishing whether an instance of the Unique Games Problem is such that it is possible to agree with a $\geq 1 - \epsilon$ fraction of the constraints or it is not possible to even agree with a ϵ fraction of them, is NP-hard.*

There is a sub-exponential time algorithm capable of distinguishing such instances of the unique games problem [ABS10], however no polynomial time algorithm has been found so far. At the moment one of the strongest candidates to break the Unique Games Conjecture is a relaxation based on the Sum-of-squares hierarchy that we will discuss below.

References

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18.S096 Topics in Mathematics of Data Science
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