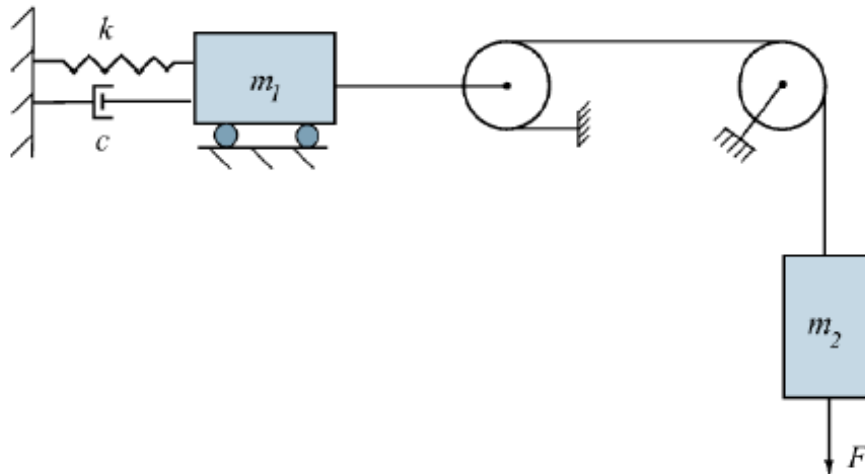


2.003/1.053 Dynamics and Controls I
Spring 2007
Problem Set 9

Issued on Wednesday, May 2nd
Due in lecture on Monday, May 7th

1 Masses and pulleys

Find the equation(s) of motion for the following system using Lagrange's equations. Ignore all friction. The pulleys have mass m and radius r and can be approximated as cylindrical in shape. The string is massless and inextensible. What is the natural frequency of the system? What is the damping ratio of the system?



2 Highway crash barriers

Highway crash barriers are designed to absorb a vehicle's kinetic energy while minimizing the deceleration of the vehicle occupants. They can be modelled as a mass-spring-damper system: the barrier's materials provide the spring constant k and the damping coefficient c and are chosen so that when a vehicle of mass m impacts them the vehicle is decelerated as safely as possible.

A particular construction has $k = 1.8 \times 10^4$ N/m and $c = 2 \times 10^4$ Ns/m. A vehicle weighing 1800 kg strikes the barrier at 50 mi per hour. Find how long it takes for the vehicle to come to rest, how far the vehicle compresses the barrier and the maximum deceleration of the vehicle. Assume that the vehicle never loses contact with the barrier.

3 Air and oil

The 2.5-kg spring-supported cylinder is set into free vertical vibration and is observed to have a period of 0.75s in part (a) of the figure. The system is then completely immersed in an oil bath in part (b) of the figure, and the cylinder is displaced from its equilibrium position and released. Viscous damping ensues, and the ratio of two successive positive displacement amplitudes is 4. Calculate the viscous damping ratio ζ , the viscous damping constant c , and the equivalent spring constant k .

