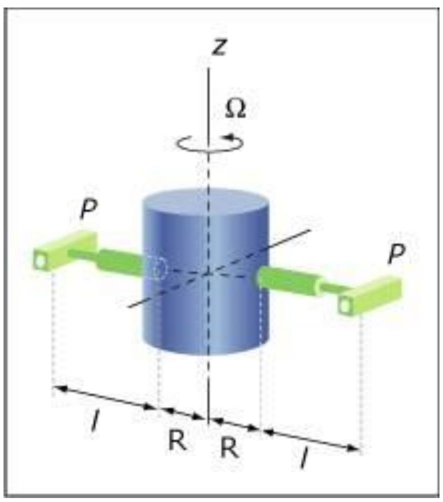
2.003 Engineering Dynamics
Problem Set 3—Problems with concept questions and answers

Problem 1:

a) A satellite is circling the earth. It spins at a constant angular rate $\Omega = 0.05(\text{rads/s})\hat{k}$. Two sensitive science instruments, indicated by 'P' in the figure, are symmetrically attached at the ends of extendable booms. Initially the booms are 1.2m in radius. Once the satellite is deployed in orbit the booms extend radially by a variable length L. As the booms are extended an internal mechanism is used to maintain the constant rotation rate of the satellite. The length L is varied from zero to 3 m. The maximum total acceleration to which the sensitive experiment modules may be subjected is 0.011 m/s^2. Determine the maximum allowable boom extension rate.



Concept Question: Does the total angular momentum of the satellite change as a result of the extension of the booms supporting the experiment modules. Assume the angular momentum is computed with respect to the center of mass of the satellite?

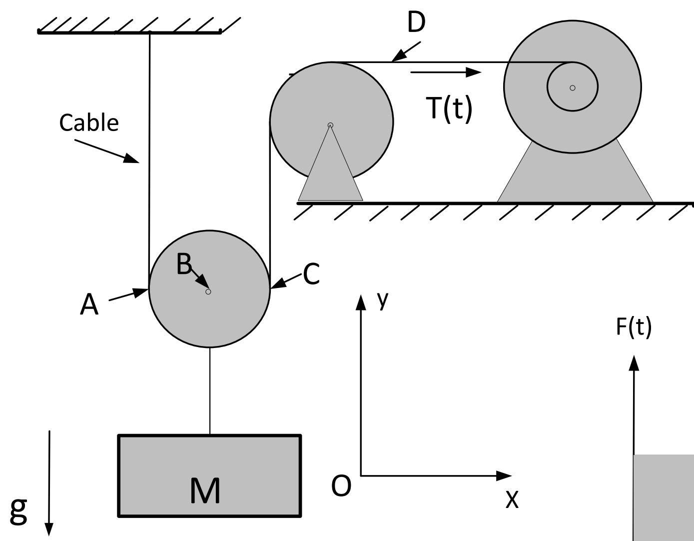
(a) Yes

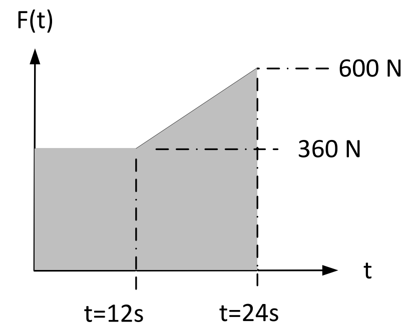
(b) No

The answer is the angular momentum does not change, because there are no external torques acting on the satellite.

Problem 2:

The winch, shown in the figure, delivers a horizontal towing force, T(t), to its cable at D. The force varies with time as shown in the graph. Determine the speed of the 80-kg mass when 't' reaches 24 s. Assume that at t=0, the mass has zero velocity. Also assume that the pulleys and cables are massless.





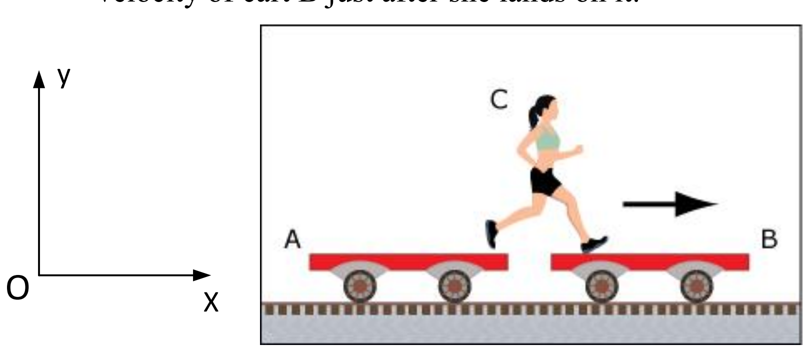
Concept question: Do you expect the speed of the mass to be constant while the force is constant during the interval 0 to 12 seconds? (a) Yes (b) No (c) I don't know what principle to apply.

The answer would likely be (b) No, because the tension in the cable is unlikely to be exactly one half the weight of the mass, which is the only condition that would result in zero acceleration.

Problem 3:

A 75-kg girl leaps from cart A with a horizontal velocity of 3 m/s measured relative to cart A. Carts A and B have the same mass of 50 kg and are originally at rest.

a. Determine the velocity of cart A just after she jumps.
b. If she then lands on cart B and comes to a stop relative to cart B, determine the velocity of cart B just after she lands on it.

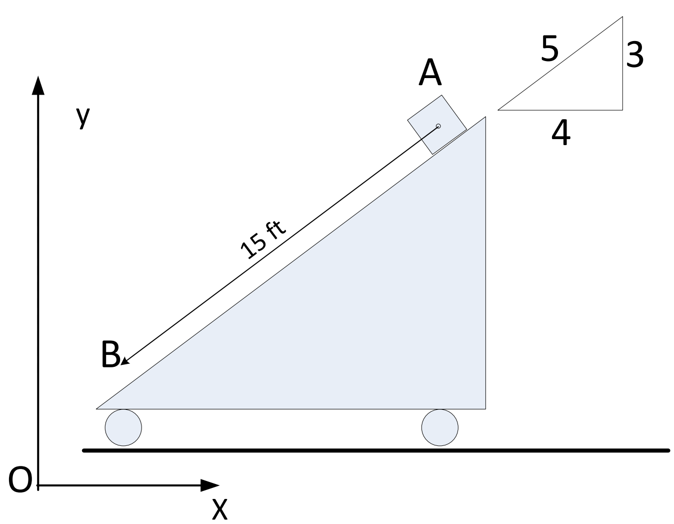


Concept question: While the girl is airborne, the relative velocity between the girl and cart A is 3 m/s. After the girl lands on the second cart, how will the relative velocity between the girl and the first cart change? (a) Increase (b) Decrease (c) I don't know what principle to apply .

Answer: In a reference frame attached to the ground the velocity of the girl will decrease, due to the requirement of conservation of linear momentum as the girl lands on cart B and sticks to it. Cart A travels at constant velocity after the girl leaps from it. It is therefore also an inertial frame. The girl will appear to slow down as she lands on cart B, as seen from cart A, because of conservation of linear momentum as seen from cart A.

Problem 4:

A ramp rolls without friction on a horizontal floor. It has a weight of 120 lb. If an 80-lb crate is released from rest at point A near the top of the ramp, determine the distance the ramp moves with respect to the floor when the crate slides 15 ft down the ramp to point B.



Concept Question:

Does the outcome of this question depend on the coefficient of friction between the weight and the ramp. Assume that, if needed, the sliding weight is brought to rest by a bumper at B. The outcome is: (a) Independent of friction on the ramp (b) Dependent on friction (c) I don't know what principle to apply

The answer is that friction between the box and ramp does not affect the final position of the ramp, because it is an internal force in the system consisting of the box and ramp together. The box does lose potential energy as it slides down the hill. The potential energy is lost as heat due to friction and in the inelastic collision of the box with the stop at the bottom of the ramp.

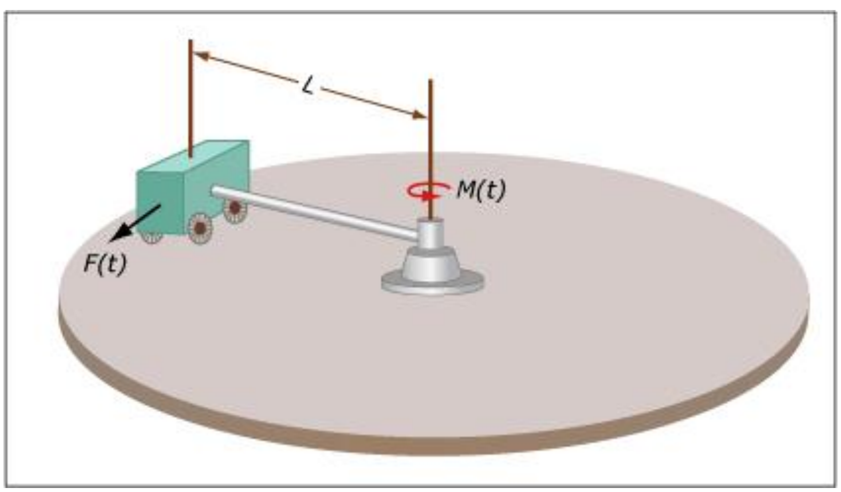
Problem 5

A massless arm rotates about a vertical axis. At the outer end of the arm is a cart. For the purpose of this problem consider the cart to be a concentrated particle. A moment(torque), M(t) is applied to the arm at the axis of rotation, given by $M(t) = 30t^2 (N-m)$. In addition an external force is applied to the mass in the tangential direction. The force is given by $F(t) = 15t(N)$. The force and moment are applied beginning at t=0.

(a) Find an expression for the angular momentum of the cart with respect to the point on the axis of rotation where the rod attaches to the pivot.

(b) At t= 5 seconds the external driving forces and moments are turned off, leaving the arm and cart to coast at a constant rotation rate. Find an expression for $\vec{P}$, the linear momentum of the cart after the force and moment are turned off.

(c) Compute the time derivative of $\vec{P}$ with respect to the Oxyz inertial frame. By Newton's 2nd law the result must be the force applied to the mass. Explain the physical meaning of this force.



Concept Question: What do you expect $\frac{d\vec{P}}{dt}$ to yield:

(a) 0 N

(b) Mg

(c) a tangential force

(d) a radial force

The answer is the radial force required to change the direction of the linear momentum, which is also known as the mass times the centripetal acceleration.

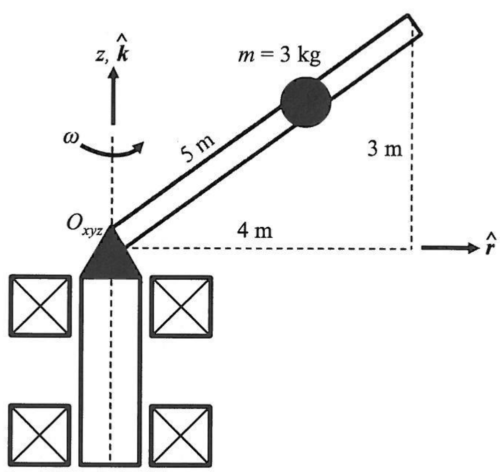
Problem 6

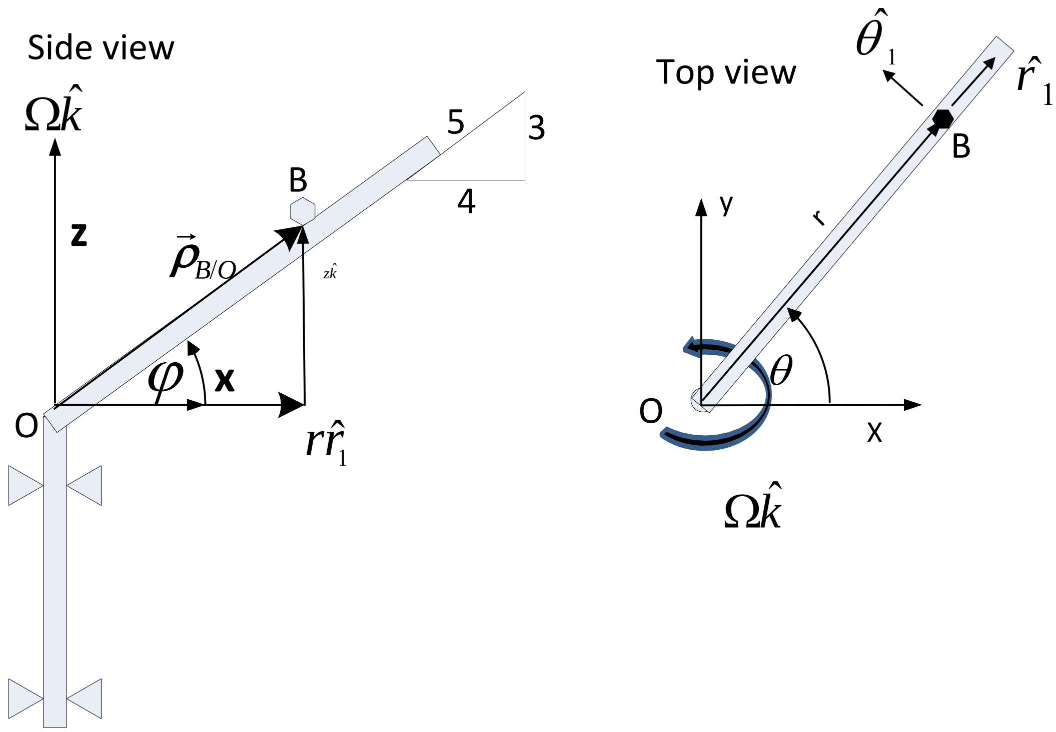
A 3 kg monkey runs up the shaft shown in the figure. The shaft rotates at a constant rate of 2.0 rad/s. The speed of the monkey running along the shaft is 1.5 m/s.

(a) Find expressions for the velocity and acceleration of the monkey with respect to a fixed inertial frame, Oxyz, located at the point the inclined shaft connects to the vertical rotating shaft.

(b) Treating the monkey as a simple particle, find an expression for the angular momentum of the monkey with respect to the origin at O. What is the direction of the angular momentum vector? Draw an arrow that begins at O to represent the angular momentum vector, $h_{/O}$.

(c)Compute the time rate of change of $h_{/O}$ with respect to the Oxyz frame. It will have three vector components or terms. Give a physical interpretation to the meaning of each term.





Concept Question: In what direction is the angular momentum vector found in part (b).

(a) z direction

(b) perpendicular to the shaft that supports the monkey

(c) parallel to the shaft supporting the monkey

The answer is that it must be perpendicular to the two terms in the cross product that define it. These are $\vec{\rho}_{B/O}$ and $\vec{P}_{/O}$, so the answer is (b), because it is in the direction $\vec{\rho}_{B/O}$.