2.003 Engineering Dynamics
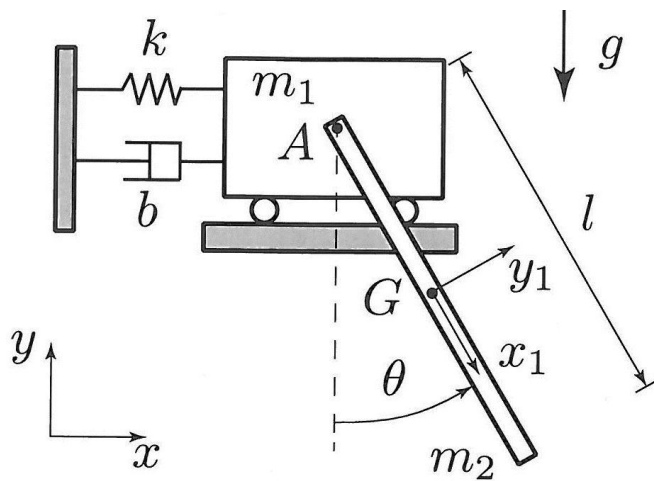# Problem Set 10

Problem 1

Figure 1. Cart with a slender rod

A slender rod of length l (2m) and mass m_2(0.5kg)is attached by a frictionless pivot at 'A' to a block of mass m_1(1kg). The block moves horizontally on rollers. The position of the block is described by the x coordinate. The block is connected to a fixed wall by a spring of constant k(10 N/m)and unstretched length l_o(0.5m),and a damper with damping coefficient b (5N-s/m). The position of the block in the inertial frame is $x\hat{i}$.

Figure 1. Cart with a slender rod

Use the results from Pset 6, #1 to find the two equations of motion of this system and linearize them. Use the linearized equations to find the undamped natural frequencies and mode shapes of the system.

Concept question: Do you expect to be able to eliminate the terms involving gravity in the equations of motion by choosing coordinates with respect to static equilibrium positions.

Problem 2

An Optics experiment is set up on a vibration isolation table which may be modeled as a single

degree of freedom system for vertical oscillations. The system has very little damping. The floor has considerable vertical vibration at 30 Hz. What must the natural frequency of the table be in order to reduce the motion of the table and optics experiment by 12 dB, when compared to the motion of the floor?
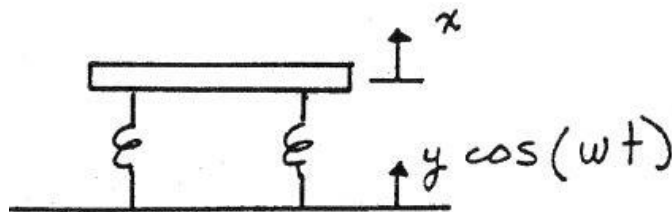
Stated mathematically this would read $-12dB = 20\log_{10}(\frac{x}{y})$. For this part of the problem

assume the damping is 0%.

When working on the table, you notice that after an accidental bump the resulting motion takes a long time to die out. To make it die down sooner you add a damper between the table and the floor, such that the total damping is 10% of critical damping. Compute the value of |x/y| with this amount of damping to that from part one.

Concept question: Will the addition of damping increase or reduce the vibration of the table in response to floor motion at 30 Hz?

Problem 3

Prof. Vandiver once measured the vibration response of a coast guard light station, which stood in 20 meters of water. It was a steel, space frame structure with a large house sitting on the top. A simple single degree-of-freedom equivalent model of the structure is a massless beam with an equivalent concentrated mass on the end. The equivalent mass is the sum of the mass of the house and 1/4 of the mass of the space frame structure. The total concentrated mass in this case was $M = 2.73*10^5$ Kg. The measured natural frequency of the platform in the first bending mode was 1.0 Hz, and the measured damping was 1.0 % of critical.

By watching the output of an accelerometer on an oscilloscope the professor was able to shift the weight of his body from one foot to the other in synchrony with the motion of the structure at 1.0

Hz. He was able to drive the structure to amplitudes in excess of that observed in a storm with 50 knot winds plus 20 foot ocean waves.
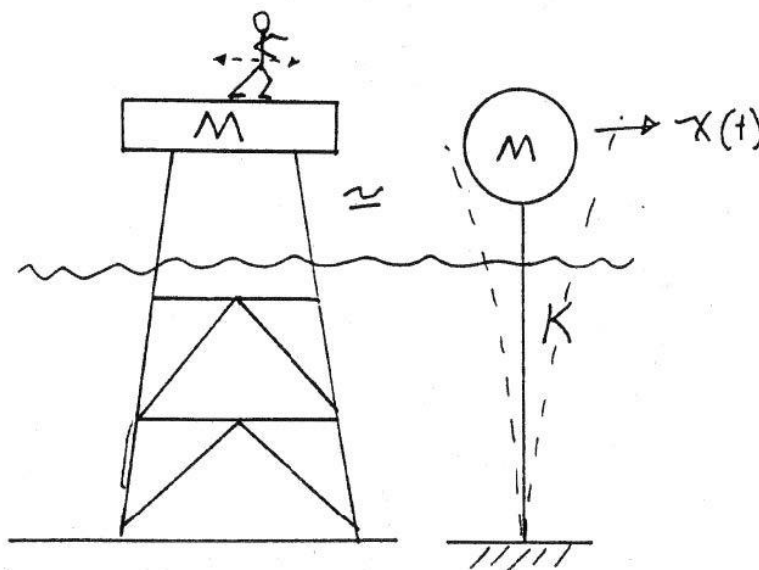
The shifting of his weight from side to side may be modeled as an horizontal harmonic force.

Find the magnitude of the steady state horizontal force that was necessary to drive the structure to a steady state acceleration

amplitude of 0.003 g's.

Concept question:

What was the steady state response amplitude expressed in meters: a). 0.003m, b). 0.003*9.81 m, c). $0.003*9.81/(6.28)^2$ m.

Problem 4

An irregularly shaped object is hinged at one point. It is supported by several springs. Its measured natural frequency is 5.0 Hz. When a static force of 10 N isapplied 0.4m from the pivot, the object rotates through an angle of 0.005 radians. What is the mass moment of inertia of the object about the pivot? Assume the body is horizontal when at rest.

Concept question: For small motions about the horizontal do you expect the natural frequency to be a function of gravity.
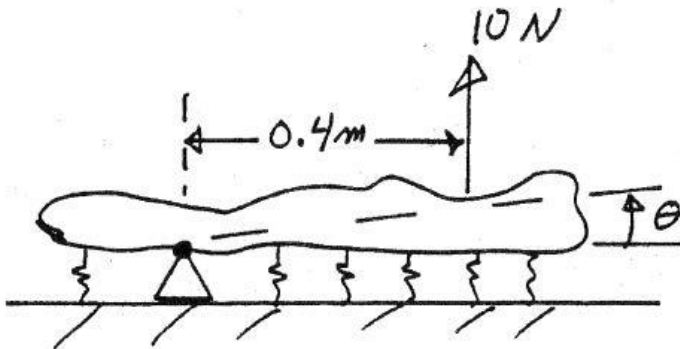
Problem 5

A 1.0 kg mass, m sits on top of a 10.0 kg mass, M. The large mass is connected to a spring, (K=250N/m) and a damper, R, and is free to oscillate horizontally on rollers. The damping ratio of the system is 5%. The coefficient of static friction between the large and small block is 0.35.

The large block is driven in steady state vibration by an harmonic force, $F_o(t)$.
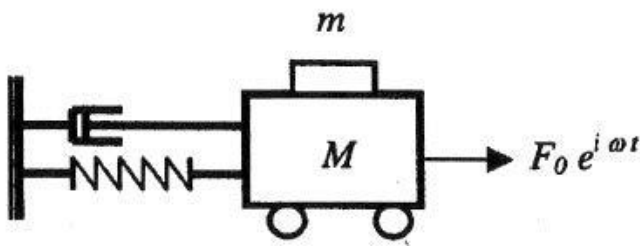
a.   What is the maximum allowable acceleration of the large block, such that the small block does not slide. Express this acceleration in

g's?

b. If the harmonic force, $F_o(t)$, is applied to the larger mass at the natural frequency of the system, what is the largest force magnitude, F_o, that may be applied such that the small mass does not slide on top of the large mass.

Concept question: When the acceleration of the system is one-half that required to make the mass slide, what is the magnitude of the friction force.

    a. μMg b. μMg / 2 c. neither

Problem 6

A mortar shell is shot almost vertically from a rail car. The rail car is spring supported as shown in the figure. It also has a damper as shown. The mass of the rail car and the mortar is M = 10,000 kg. The mass of the shell is 25 kg. The velocity of the shell is 150m/s as it leaves the mortar. The spring stiffness of the suspension system of the rail car is k =15,791 N/m. The damper constant, R, is 251.33 Ns/m.

$V_0 = 150$ m/s

m

$x(t)$

k R
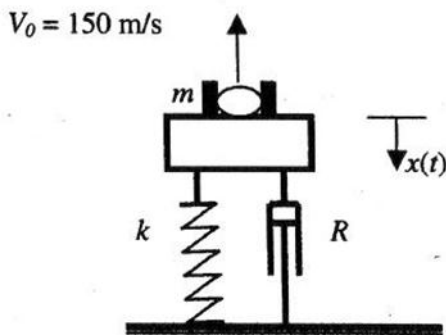
a. Write down the equation of this single degree of freedom system for vertical motion. Determine the natural frequency and damping ratio (obtain numerical values).

b. The mortar is loaded and fired when the system is initially at rest (no motion). After firing the shell the car will vibrate. Find an expression for x(t), the time history of the vibration of the car, after the shell has been fired. Be specific about the initial conditions that you use. Sketch the result.

c. The vibration of the car will decay with time after firing. Predict the ratio of two vibration amplitude peaks separated by five periods of vibration.

Concept question: What initial conditions will be required? A. initial displacement only, B. Initial velocity only, C.Both initial velocity and displacement.

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