2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*
Spring Term 2008

Problem Set 7

Assigned: April 4, 2008 Due: April 11, 2008

Reading:

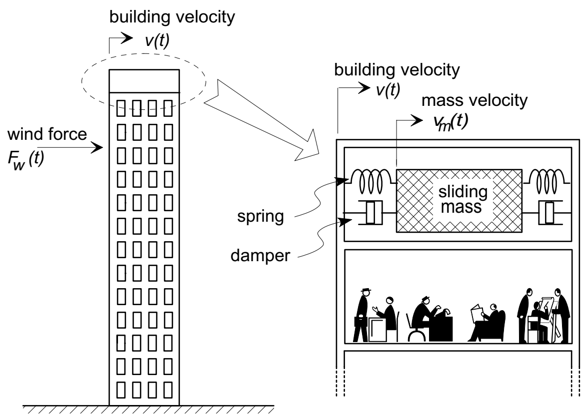
- Nise: 4.1 – 4.8

Problem 1: Preparation for the Final Lab Project: Labs 8 – 10

> You may use any method you like to solve this problem, including any software tools you choose. You must show your method...

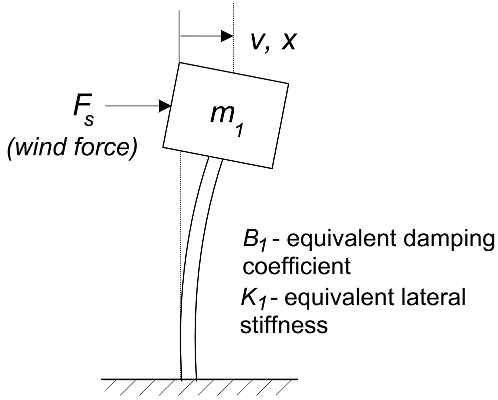
Modern high-rise buildings often suffer from wind-induced motion problems. The tall, slender aspect ratio means that they are resonant flexible structures with little damping, and the resulting motion during high winds can induce motion sickness and fatigue for the occupants, as well as the risk of structural damage.

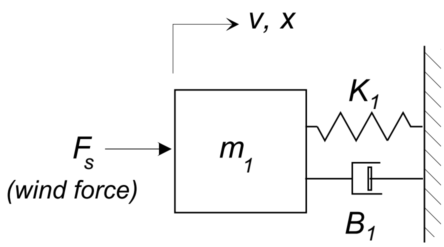
Architects and structural engineers often address these issues by incorporating active or passive *tuned-mass dampers* into the building structure. For example, the John Hancock tower across the river in Boston has two 300 ton sliding masses at the top of the building to help suppress wind-induced motion.



In the final three lab sessions you will design and implement an active damping system and test it on a model building. To prepare for this project we will develop linear models of the building with a damper in this assignment, and you will use the results as the basis for your control system design in the lab.

A lumped model of an undamped building is shown below, where the building mass m_1 is concentrated at the top of a tall slender column that acts as a lateral spring with equivalent stiffness K_1. (Of course, in a real building the mass is distributed along the length of the column, but we use the simplifying assumption of concentrated mass - and our experimental set-up closely follows this assumption.) For small displacement angles we assume that the motion of the mass is translational, and that the air resistance and structural damping may be modeled as a small but finite equivalent viscous friction coefficient B_1.

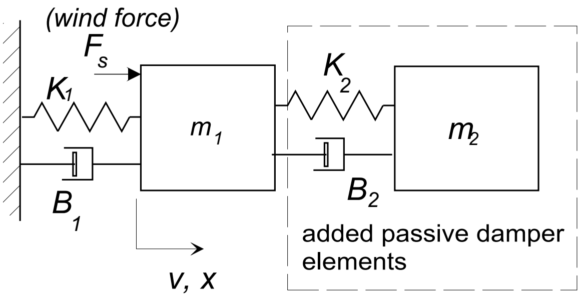




This simplified model is the classic parallel mass-spring-dashpot system, excited by an external force F_s, and has a transfer function relating the velocity v to the force

$$G(s) = \frac{V(s)}{F_s(s)} = \frac{s}{m_1 s^2 + B_1 s + K_1}$$

A passive tuned-mass damper adds additional elements to the system as shown below.



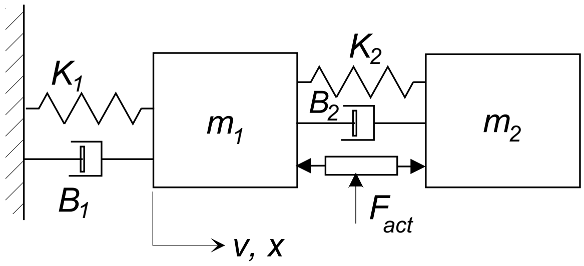
The values of m_2, B_2, and K_2 are chosen so that the motions of the two masses are *out-of-phase*, that is when m_1 is travelling to the right m_2 is travelling to the left. The forces transmitted through K_2 and B_2 act to inhibit the motion of m_1. Effective reduction of the motion requires careful "tuning" of the elements.

The figure above represents the experimental apparatus used in the lab project. In order to design an active tuning system you will need a model of this system.

(a) Derive the transfer function relating the velocity v of m_1 to the wind force F_s.

Hint: The simplest way (I think) is to combine all of the elements into an equivalent impedance. Your model should turn out to be fourth-order, with really messy coefficients...sorry about that!

In the lab you will be using active control to "tune" the damping system. We will provide sensors that measure the motion of m_1 and a force actuator F_{act} between the two mass elements as shown below.



(b) Derive the transfer function relating the velocity v of m_1 to the controller actuator force F_{act}. (Assume the wind force is zero.)

Hint: Your transfer function denominator should turn out to be the same as found in part (a) above.

Make a copy of your solution and bring it to lab in the week of April 14.

You may need to generate additional transfer functions during the course of the project.

Problem 2: Nise Problem 4-23 (p. 207).

Problem 3: Nise Problem 4-29 (p. 208).

Problem 4: Nise Problem 4-55 (p. 212).

Problem 5: Nise Problem 4-62 (p. 214).

Problem 6: Nise Problem 4-67 (p. 215).