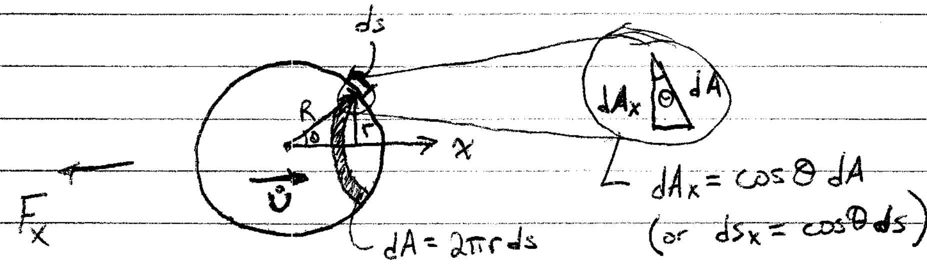
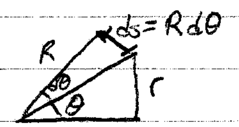
2.016 HW #3 Fall 2005

1a)





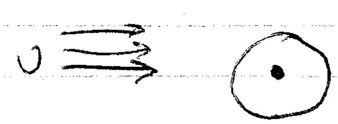
Body moving to the right with velocity U and acceleration $\dot{U}$, so force is to the left (which is why we get the negative sign at the end)

$$F_x = \int_0^{\pi} [\text{dynamic pressure}] \cdot \cos\theta \; 2\pi (R\sin\theta)\, R\, d\theta$$

integrate $\theta = 0$ to π to cover entire surface.

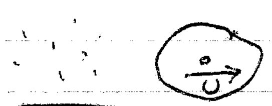
b) The dynamic pressure changes as you go around the sphere. The static pressure also changes as you go around the sphere, but if you integrate that around the whole thing, you just get the buoyancy force (try it.) so that's not interesting. In dynamic problems (added mass, drag, etc), we work with the dynamic pressure.

c) Fluid moving & stationary sphere



$\phi = U\cos\theta\left(r + \frac{R^3}{2r^2}\right)$ $\quad F_x = +\rho \frac{6}{3}\pi R^3$

quiescent fluid & moving sphere



$\phi = U\cos\theta\left(\frac{R^3}{2r^2}\right)$ $\quad F_x = -\rho \frac{2}{3}\pi R^3$

d) true. Free stream flow $\quad U \Rightarrow$ $\quad \phi = \underbrace{U\cos\theta \cdot r}_{Ux}$ $\quad$ "$\underbrace{F_x = \rho \frac{4}{3}\pi R^3 = \rho \forall}_{\text{left over}}$"

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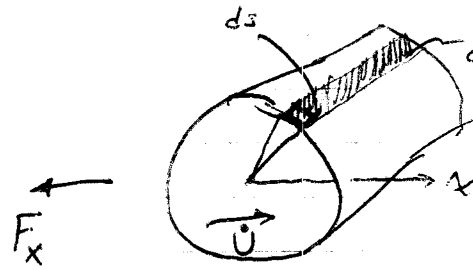
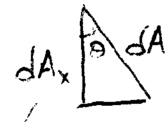
1 e)



$dA = L\,ds$
$= LR\,d\theta$



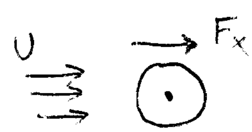
↳ $dA_x = dA\cos\theta$

$$F_x = \int_0^{2\pi} [\text{dynamic pressure}]\cos\theta\, RL\, d\theta$$

↳ integrate $\theta = 0$ to 2π to go all the way around

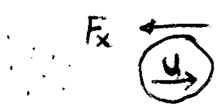
f) dynamic pressure

g) Fluid moving & stationary cylinder



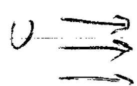
$\phi = U\cos\theta\left(r + \frac{R^2}{r}\right)$ $F_x = 2\rho\pi R^2 L$

(−) quiescent fluid & moving cylinder



$\phi = U\cos\theta\left(\frac{R^2}{r}\right)$ $F_x = -\rho\pi R^2 L$

Free stream Flow



$\phi = U\cos\theta\, r$
$= Ux$

"$F_x = \rho\pi R^2 L$"
$= \rho\forall$

h. true.

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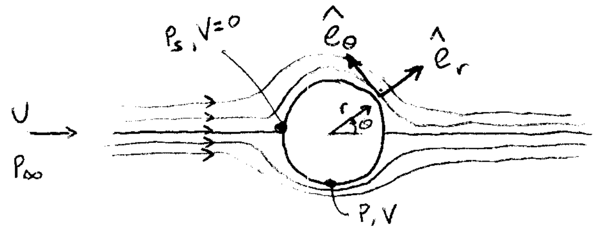
2.



a) $\phi = U\cos\theta\left(r + \frac{R^2}{r}\right)$

$$\vec{V} = \frac{\partial\phi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{e}_\theta = \underbrace{U\cos\theta\left(1 - \frac{R^2}{r^2}\right)}_{\text{velocity in r-direction}}\hat{e}_r - \underbrace{U\sin\theta\left(1 + \frac{R^2}{r^2}\right)}_{\text{velocity in }\theta\text{-direction}}\hat{e}_\theta$$

b) Bernoulli's: $P + \frac{1}{2}\rho V^2 = \text{constant}$

$$P + \frac{1}{2}\rho V^2 = P_\infty + \frac{1}{2}\rho U^2, \qquad V = 0\,\hat{e}_r - 2U\sin\theta\,\hat{e}_\theta \text{ at } r=R$$

$$P + \frac{1}{2}\rho\cdot 4U^2\sin^2\theta = P_\infty + \frac{1}{2}\rho U^2$$

$$\underline{P = P_\infty + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)}$$

In terms of the stagnation pressure:

$$P + \frac{1}{2}\rho V^2 = P_s + 0$$

$$P = P_s - \frac{1}{2}\rho\cdot 4U^2\sin^2\theta$$

c) From Bernoulli's $P - P_\infty = \frac{1}{2}\rho U^2 - \frac{1}{2}\rho V^2$

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho U^2} = \frac{\frac{1}{2}\rho U^2 - \frac{1}{2}\rho V^2}{\frac{1}{2}\rho U^2} = 1 - \frac{V^2}{U^2}$$

d) $$C_p = 1 - \frac{4U^2\sin^2\theta}{U^2} = 1 - 4\sin^2\theta \quad \text{(see chart)}$$

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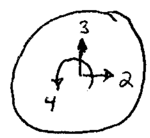
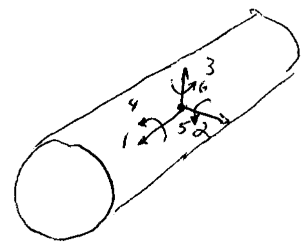
3.

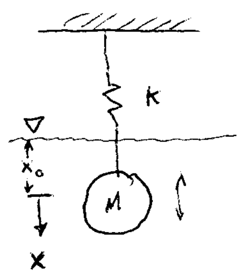




$$\underline{M_{33}} = \rho \pi R^2 L = 1000 \tfrac{kg}{m^3} \cdot \pi \cdot (0.01m)^2 (1m) = \underline{0.314\ kg}$$

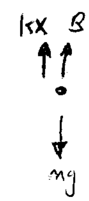
$$\underline{M_{44} = 0}$$

4



Hydrostatically

Dynamically



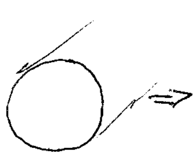
$$B = mg$$

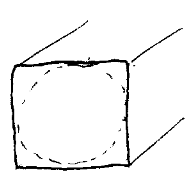
$$\Sigma F = ma = \underbrace{m}_{M+m_a}\ddot{x} = \underbrace{mg - B}_{0} - kx$$

$$\boxed{(M+m_a)\ddot{x} + kx = 0}$$

Note: If there were some spring force hydrostatically, it would sum to zero here anyway. The kx part is the only important part dynamically.

b)





Since the square has a larger cross section (i.e. more volume) and since it is less streamlined, I would expect the added mass to be <u>more</u>. Therefore, the natural frequency would go <u>down</u>.

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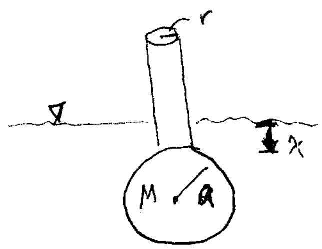
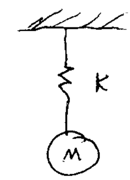
5.



Think of this as a spring-mass system, with the Buoyancy force increasing linearly with depth:



$$B = \rho \forall_{sphere} g + \rho \forall_{cylinder} g = \rho \forall_{sphere} g + \rho \pi r^2 x g = \rho \forall_{sphere} g + \underbrace{(\rho g \pi r^2)}_{K} \cdot x$$

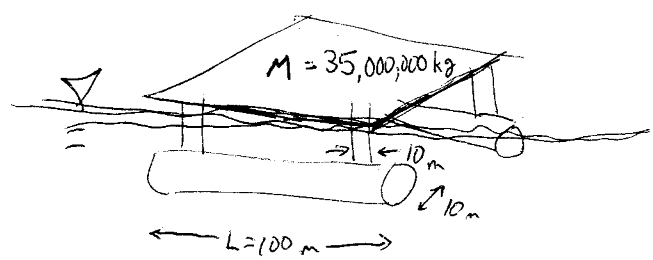
$$M_a = \rho \tfrac{2}{3} \pi a^3$$

$$(M + m_a)\ddot{x} + kx = 0$$

$$\boxed{\left(M + \rho \tfrac{2}{3}\pi a^3\right)\ddot{x} + (\rho g \pi r^2)\, x = 0}$$

$$\omega = \sqrt{\frac{K}{m}} = \boxed{\sqrt{\frac{\rho g \pi r^2}{M + \rho \frac{2}{3}\pi a^3}}}$$

6. $\boxed{(M + M_a)\ddot{x} + kx = 0}$



$$M_a = 2 \cdot \rho \pi \cdot (5\,m)^2 \cdot 10\,m = 1{,}570{,}000 \text{ kg}$$

$$k = 4 \cdot \rho g \pi (5\,m)^2 = 314{,}000 \ \frac{N}{m}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{314{,}000}{35{,}000{,}000 + 1{,}570{,}000}} = 0.09 \ \frac{rad}{s} \cdot \frac{1 \text{ cyc}}{2\pi \text{ rad}} = \underline{0.015 \text{ Hz}}$$