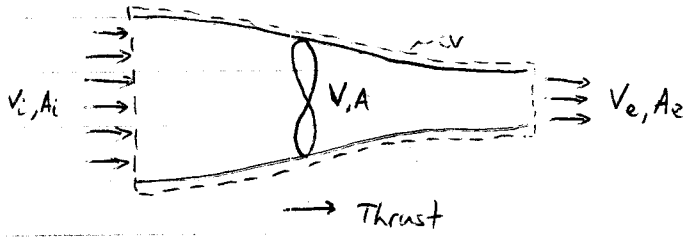
2.016 Homework #8 Solutions

1)



Assumptions:

- Ambient pressure acts everywhere on CV $\Rightarrow$ no net force
- steady flow
- incompressible, invicid flow
- uniform flow at each section

a) Conservation of Mass $\quad \dot{m} = \rho V_i A_i = \rho V A = \rho V_e A_e$

Conservation of Momentum $\quad -\dot{m} V_i + \dot{m} V_e = T$

$$T = \rho V A (V_e - V_i)$$

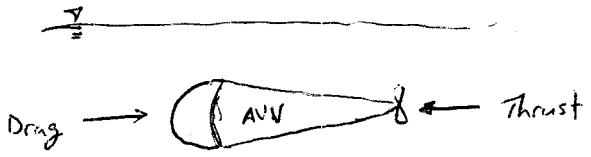
b) Power = Force · Velocity

Power at propeller $= T \cdot V$

c) $$\text{efficiency} = \frac{\text{useful Power}}{\text{power consumed}} = \frac{\text{Thrust} \cdot \text{boat velocity}}{\text{thrust} \cdot \text{velocity at propeller}} = \frac{T V_i}{T V} = \frac{V_i}{V}$$

d) It is always <u>less</u> than one, because $(V > V_i)$ the fluid accelerates from the inlet to the propeller (and continues to accelerate to the exit). Recall $V = \frac{V_i + V_e}{2}$

HW # 8 Solution

a) 

At steady speed, thrust = drag

$$D = \frac{1}{2}\rho U^2 A C_D$$

C_D is a function of the Reynolds Number

$$Re = \frac{UD}{\nu}$$

To have "geometric similarity" make a scale model with all dimensions proportional to the full-size ship. To have "dynamic similarity" when the forces behave similarly, the Reynolds number must be the same for the model as it is for the full-scale AUV.

a) $$Re_{AUV} = \frac{3\,m/s \cdot D_{AUV}}{10^{-6}\,m^2/s} = \frac{U_{model} \cdot \frac{D_{AUV}}{10}\,m}{10^{-6}\,m^2/s} = Re_{model}$$

$$U_{model} = 30\,m/s$$

b) $$D_{model} = 10\,N = \frac{1}{2}\rho (30\,m/s)^2 \cdot \frac{\pi \left(\frac{D_{AUV}}{10}\right)^2}{4} C_D = \frac{1}{2}\rho (3\,m/s)^2 \cdot \frac{\pi D_{AUV}}{4} C_D = D_{AUV}$$

$$D_{AUV} = 10\,N$$

Note: Assuming $C_D = 0.5$

$$D_{model} = \frac{1}{2} \cdot 1000 \cdot 30^2 \cdot \frac{\pi (0.175)^2}{4} \cdot 0.5$$
$$= 950\,N$$

c) Re, C_D