PART A —

(1)

X	0	0	0	0	X
0	X	0	0	0	0
0	0	X	X	0	0
0	0	X	X	0	0
0	0	0	0	0	0
X	0	0	0	0	X

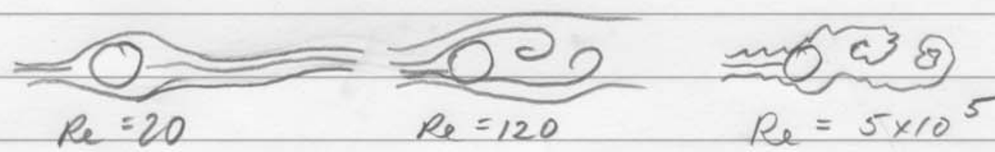
(3)

−X	0	0	0	0	−X
0	−X	0	0	0	−X
0	0	−X	+X	+X	0
0	0	+X	−X	0	0
0	0	+X	0	−X	0
−X	−X	0	0	0	−X

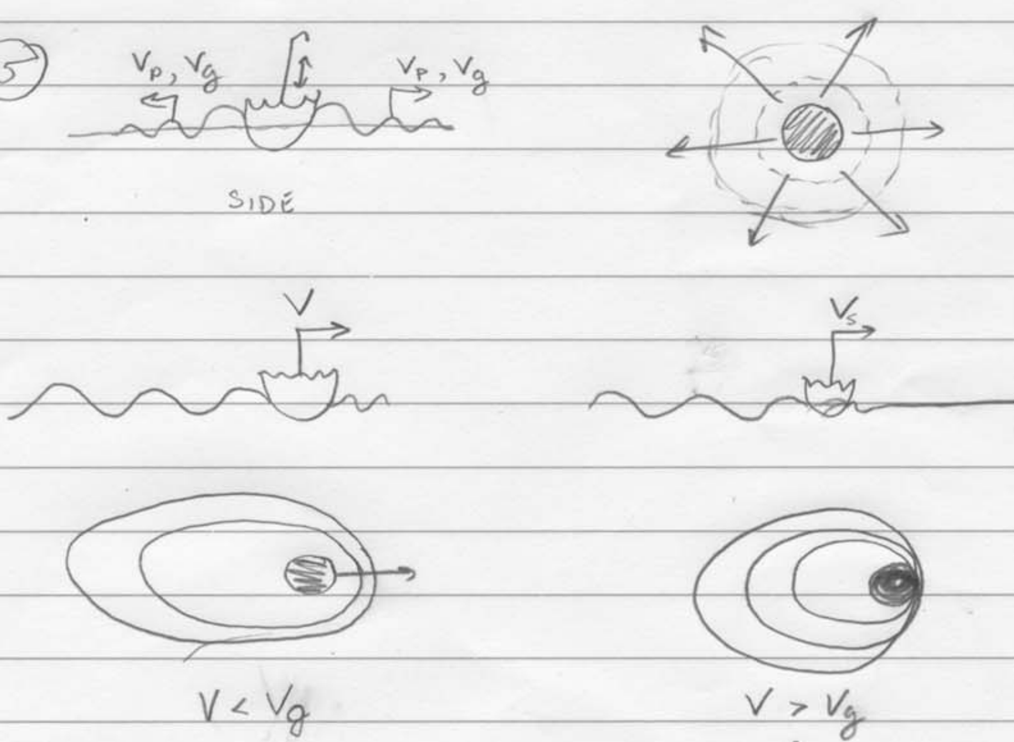
(2) $\underline{\underline{\omega_s}} = 16.75 \text{ rad/s}$, $(f_s = 2.67 \text{ Hz})$

$$\omega_s = \omega_n = \sqrt{\frac{k}{M + M_a}}$$ lock in —

(4)

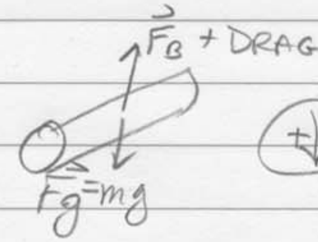


(5)



there is always some form of bow wave...

PART B-



(1) a) Terminal Velocity

$(+\downarrow)$

$F_{DRAG} = \frac{1}{2}\rho U^2 C_D A$

$F_B = \rho_w g \forall$ $\quad\quad A = dL$

$F_g = 1.5\rho_w g \forall$ $\quad\quad \forall = \frac{\pi d^2}{4} L$

@ Terminal Vel $\Sigma F = ma = 0$

$$\Sigma F = \vec{F_g} - \vec{F_B} - \vec{F_D} = 0$$

$$\vec{F_D} = \vec{F_g} - \vec{F_B} = \frac{1}{2}\rho_w g \forall = 157N$$

$$u^2(\cancel{\tfrac{1}{2}}\cancel{\rho_w} C_D \cancel{d}\cdot\cancel{L}) = \cancel{\tfrac{1}{2}}\cancel{\rho_w} g \frac{\pi d^{\cancel{2}}}{4}\cancel{L}$$

$$u = \sqrt{\frac{\pi g d}{4 C_D}} = \sqrt{1.57/C_D}$$

laminar $C_D = 1.2$ $u = 1.14\ m/s$ $Re = 228{,}000$

turb $C_D = 0.6$ $u = 1.62\ m/s$ $Re = 324{,}000$

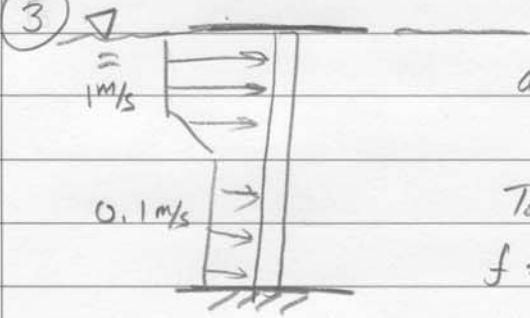
wow! v. close could be either so it ultimately depends on cylinder roughness - either answer gets credit -

b) @ $u = 1.14\ m/s$ $f = 1.14\ hz$; $f = \frac{S u}{d} = \frac{0.2}{0.2}u$

$u = 1.62\ m/s$ $f = 1.62\ hz$

PART B

(3)



assume $S = 0.2 = \frac{fd}{u}$

Top

$f = \frac{0.2 \cdot 1.0}{0.5}$

$f_{top} = 0.4\ Hz$

bottom

$f_{bot} = 0.1 * f_{top}$

$f_{bot} = 0.04\ Hz$

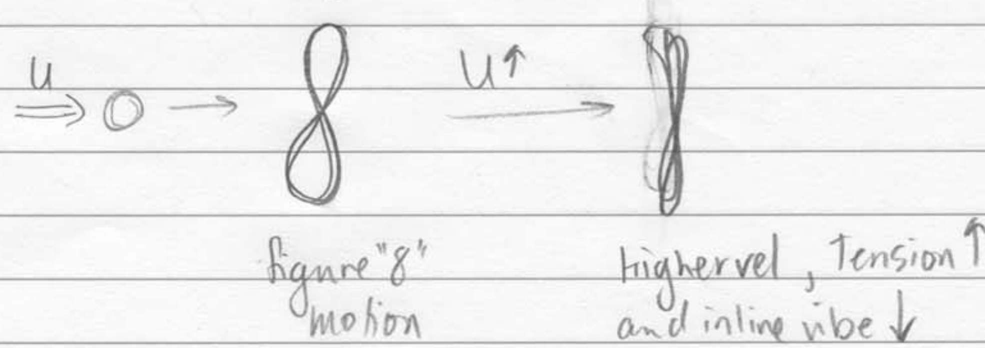
b) frequency of drag is 2·freq of vortex shedding

freq. of lift is = freq of vortex shedding

therefore it directly correlates w/ flow velocity.

c) The structure will tend to vibrate more at top exciting forced motion @ bottom. In Both cases frequency is Low so it's not going to be too violent. Shear will complicate shedding & make flow more 3 dimensional

d) Flexible cylinder



PART B –

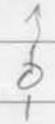
(2) Initial Acceleration

$$\Sigma F = M\ddot{x}_3\Big|_{0=t} = \vec{F}_g - \vec{F}_g$$ only forces at work!

$M = m + m_a \qquad m = \rho \forall = 2\rho_w \forall$

$$\forall = \forall_{sphere} + 2\forall_{cylinders}$$
$$= \frac{4}{3}\pi(2a)^3 + 2\cdot\pi\left(\frac{a}{2}\right)^2(4a)$$
$$= \frac{32}{3}\pi a^3 + 2\pi a^3 = \boxed{\frac{38}{3}\pi a^3 = \forall}$$

(i) vertical



$$m_a \cong M_{a\,sphere\ only} + \underbrace{\cancel{\varnothing}}_{\text{negligible compared to sphere}}$$
$$= \tfrac{1}{2}\rho_w \forall_s = \frac{16}{3}\rho_w \pi a^3$$

(↓+) $(m+m_a)\ddot{x}_3 = mg - \rho_w g \forall$
$$= 2\rho_w g\forall - \rho_w g \forall$$
$$\ddot{x}_3 = \frac{\rho_w g \forall}{m + m_a}$$
$$= \frac{\cancel{\rho_w} g \frac{38}{3}\cancel{\pi a^3}}{\frac{76}{3}\cancel{\rho_w \pi a^3} + \frac{16}{3}\cancel{\rho_w \pi a^3}}$$

take as convention

(accel is + down) $\ddot{x}_3 = \dfrac{38g}{92} = \dfrac{19}{46} g \ m/s^2$

lower added mass so slightly larger initial acceleration

$\ddot{x}_3 = 4.13\, m/s^2$ (↓+)

(ii) horizontal

$$m_a\Big|_h = \tfrac{1}{2}\rho\forall_s + \underbrace{2\pi\left(\frac{a}{2}\right)^2(4a)\rho_w}_{\text{cylinders}}$$
$$m_a\Big|_h = \frac{22}{3}\pi\rho_w a^3$$

$(m+m_a)\ddot{x}_3 = mg - \rho_w g\forall = 1\rho_w g \forall$

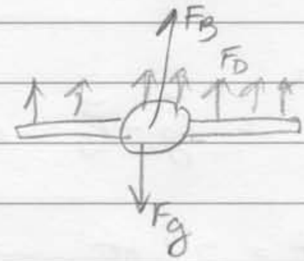
$$\ddot{x}_3 = \frac{\cancel{\rho_w} g \left(\frac{38}{3}\cancel{\pi a^3}\right)}{\left(\frac{76}{3} + \frac{22}{3}\right)\cancel{\rho_w \pi a^3}}$$
$$= \frac{38}{98} g$$

$\ddot{x}_3 = \dfrac{19}{49} g \ \dfrac{m}{s^2}$ (down +)

slightly lower due to higher added mass

$\ddot{x}_3 = 3.88\, m/s^2$ (↓+)

(b)



Terminal Vel $\quad \Sigma F = 0 = m\ddot{x}_3 \Rightarrow \ddot{x}_3 = 0$ no added mass per se.

$$mg - \rho g \forall - \tfrac{1}{2}\rho U^2 C_D A = 0$$

$$\rho g \forall = \tfrac{1}{2}\rho U^2 (C_D A) \qquad u^2 = \frac{\rho g \forall}{\tfrac{1}{2}\rho C_D A} = \frac{2 g \forall}{(C_D A)}$$

$$(C_D A) = A_{sphere} C_{D sphere} + 2 A_{cyl} C_{D cyl}$$

$$= 4\pi a^2 \cdot C_{D_s} + 2a(4a) C_{D_c}$$

$$U = \sqrt{\frac{\cancel{2} g \cdot \frac{38}{3}\pi a^{\cancel{3}}}{\cancel{2}4 \cancel{a^2}(\pi C_{D_s} + 2C_{D_c})}} = \sqrt{\frac{19 g \pi a}{3(\pi C_{D_s} + 2C_{D_c})}}$$

if a small $(a \to 0)$ $\quad U \to 0$

a large then $U \doteq \sqrt{\frac{19 g \pi a}{3(\pi C_{D_s} + 2C_{D_c})}}$

($C_{D_?}$ chosen based on Re#)

(c) $M_1 = ? \; -\varepsilon_{jkl} u_i u_k m_{li}$

$M_2 = ?$

$\boxed{M_3 = 0}$ by symmetry

$u = (0, -u_2, u_3, 0, 0, 0)$

no accelerations on angular velocities!

Munk moments only –

$M_1 = -\overset{(+1)}{\varepsilon_{123}} u_2 u_3 m_{33} - \overset{(-1)}{\varepsilon_{132}} u_3 u_2 m_{22}$

$j = 1$

$(k = 2, 3)$ 2 → $i = 3$ only

$i = 2, 3$ 3

$l = 1-3$ 3

$m_{li} =$ 12 13 / 22 23 / 32 33

$1 \to 2$, $+$, 3

$m_{11}, m_{22}, m_{33} \neq 0$

$m_{55}, m_{66} \neq 0$

Rest are zero!

$$\boxed{m_1 = -(u_2 u_3)[m_{22} - m_{33}]} = 0$$

$j = 2 \Rightarrow$ $\boxed{m_2 = \emptyset}$

$-\varepsilon_{2kl}$

$j = 2$

$k = 3$ only

$l = 1$ then

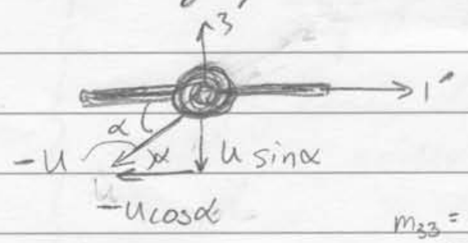
$i = 1$ only

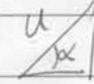
but $u_1 = 0 \; \therefore$

b/c $m_{22} = m_{33}$ by symmetry!

for $j = 3$ then $k = 2$ only, $l = 1$ only so $i = 1$ only $u_1 = 0 \; \therefore m_3 = 0$

(d) munk moment only





$U = (-U\cos\alpha, 0, U\sin\alpha, 0, 0, 0)$

$m_{33} = M_{a\perp}$ from part a

$m_{11} = M_{av}$ from pt a.

Want $M_2 = -\varepsilon_{213} U_1 U_3 m_{33} - \varepsilon_{231} U_3 U_1 m_{11} = U_1 U_3 [m_{33} - m_{11}] \neq 0$

$j = 2 \quad k = 1, 3 \quad i = 1, 3 \quad l = 1, 3$ for nonzero m_{il}