11 Sea Spectrum and Marine Vehicle Pitch Response

1. Make a plot of the spectrum for about one hundred frequencies from zero to 4 rad/s, with modal frequency $\omega_m = 1$ rad/s, and significant wave height $H_{1/3} = 0.90$m.

   See the top graph in Figure 2 for the wave spectrum.

2. Confirm that the area under the spectrum is equal to E_S, by making a numerical integration. You can then take this E_S to double-check $H_{1/3}$.

   The area under the curve E is 0.0502; it was supposed to be 0.0506. Note in the MATLAB code that I randomize the actual frequencies used; this is to avoid related frequencies (as seen in the first problem above). The corresponding significant height is 0.896m; pretty close to the desired value of 0.90m.

3. Make ten minutes worth of this wave-like data, using a sampling period of 0.1 seconds, and show a plot, with the original $H_{1/3}$ maximum and minimum levels indicated.

   See the middle graph in Figure 2 for a time-domain realization of this spectrum.

4. Compute the parameter E_Y from the area under $Y(\omega)$, and so estimate the "significant height" of the pitch motion.

   The significant height of the pitch motion is 0.26 radians, or about fifteen degrees. The lower graph in the figure shows the multiplication in frequency space.

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%---------------------------------------------------------------------------
%  Bretschneider Sea Spectra and Vehicle Pitch Response

clear all;

dfreq = .04;         % frequency resolution for creating waves, rad/s
maxfreq = 3.99 ;     % highest wave frequency made, rad/s
dt = .1 ;            % sampling period, s
tff = 600 ;          % final time, s
wmodal = 1 ;         % modal frequency (used for Bretschneider construction)
E = .05 ;            % spectra parameter for Bretschneider:
                     % E = sig_height^2 / 16

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% no user parameters below this point
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

time = 0:dt:tff;         % time vector
n = length(time) ;       % number of samples
freq = dfreq:dfreq:maxfreq;             % imposed frequencies
                                        % (near zero to maxfreq)
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freq = freq + (.5-rand(1,length(freq)))*dfreq ;
        % add random components in freq vector

disp(sprintf('Imposed dfreq/available resolution: %g (best if below one)',...
    dfreq/(2*pi/dt/length(time)))) ;

if (pi/dt < max(freq)),
    disp('Must have higher sampling rate to avoid aliasing! -- ABORT.');
    break ;
end;

% make up a Bretschneider spectrum and then get out the amplitudes
% for the example case
B = wmodal^4*1.25 ;
H = 4*sqrt(E) ;
A = 4*B*E ;
sBret = A./freq.^5.*exp(-B./freq.^4);

figure(1);clf;hold off;
subplot('Position',[.2 .2 .5 .2]);
plot(freq,sBret,'LineWidth',2);
xlabel('rad/s');
title('Bretschneider Spectrum, H_{1/3} = 0.90m');
print -deps bret_veh1.eps

disp(sprintf('E:  %g vs. %g.', E, sum(sBret)*dfreq));

amp = sqrt(2*sBret*dfreq) ;
% Note that sBret is invariant with dfreq, but amp definitely changes
% with dfreq, according to PNA def. of spectrum given in the problem.

% make up the time series, with random phase
phase = 2*pi*randn(length(freq),1);
x = zeros(size(time));
for i = 1:length(freq)
    x = x+amp(i)*cos(freq(i)*time+phase(i));
end

figure(2);clf;hold off;
subplot(211);
plot(time,x); xlabel('Time, seconds')
hold on;
plot([0 tff],[1 1]*H/2,'r--',[0 tff],-[1 1]*H/2,'r--');
legend('Simulation','+/- H_{1/3}/2');
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print -deps bret_veh2.eps

% Here is the transfer function, and the pointwise frequency multiply of
% F*F' with S
i = sqrt(-1);
F = (.4*i*freq + .3) ./ (-freq.^2 + i*freq*1 + 3) ;
FF = F.*conj(F) ;

figure(3);clf;hold off;
subplot('Position',[.2 .2 .5 .5]);
plot(freq,FF,freq,sBret,'--',freq,FF.*sBret*20,':','LineWidth',2) ;
legend('FF*','S','20 x S x FF*',1);
xlabel('rad/s');
print -deps bret_veh3.eps

% Compute the area under the curve and from it get the significant
% response height
FFS = FF.*sBret ;
EFFS = sum(FFS)*dfreq ;
FFSsig = 4*sqrt(EFFS) ;
disp(sprintf('Significant response height:  %g.', FFSsig));

%-------------------------------------------------------------------------
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