15 Bouncing Robot

You are asked to explore the simple dynamics of a bouncing robot device, using simulation. There is a single mass with a very light helical spring attached on the bottom of it; the mass is $40kg$, and the spring constant is $10400N/m$. The spring additionally has a little bit of damping, $35Ns/m$. The spring is NOT attached to the ground. The initial condition has zero vertical velocity and a $15cm$ compression of the spring.

1. Formulate the system dynamics with an annotated drawing and a statement of the governing equations.

   Solution: Let us say that vertical position $y = 0$ corresponds with the spring in its natural, uncompressed state, and it is just touching the ground. Then when $y < 0$, there is a compression force pushing the mass upward at ky, and the associated damping $b\dot{y}$. When $y > 0$, there is no spring force at all. In all cases, there is a steady gravity force acting. We have then:

$$\begin{aligned} m\ddot{y} + b\dot{y} + ky &= -mg \text{ when } y < 0 \text{ , or} \\ m\ddot{y} &= -mg \text{ when } y > 0. \end{aligned}$$

2. Is this a linear system - why or why not? Since we are not forcing the system, you can answer the question by considering how the response changes as you scale the initial conditions.

   Solution: The system is nonlinear. If we apply a very large preload before release, we will get a large hopping response, whereas if we apply a very small preload the mass may not hop at all, because the spring doesn't fully unload.

3. Run a simulation to get the response up to time ten seconds. Show in a plot both the vertical position and the velocity as a function of time.

   Solution: See that attached code and figure. Note that when the spring is unloaded, the velocity is changing linearly with time (constant acceleration), whereas when the spring is loaded, a sinusoidal velocity profile occurs.

4. Answer the specific question: At what time after the release does the lower end of the spring stop leaving the ground?

   Solution: The spring does not leave the ground again after 3.48 seconds.

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Simulate a bouncing mass

function bouncingMass

clear all;

[t,y] = ode45(@func,[0 10],[0 -0.15]) ;

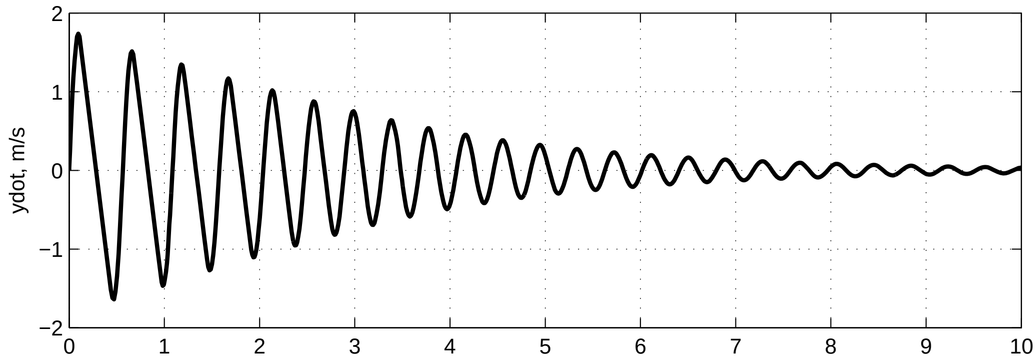
figure(1);clf;hold off;
subplot(211);
plot(t,y(:,1),'LineWidth',2); grid;
ylabel('ydot, m/s');
subplot(212);
plot(t,y(:,2),'LineWidth',2); grid;
ylabel('y, m');
xlabel('time, seconds');

function [ydot] = func(t,y) ;
                % all units MKS
m = 40 ;        % mass
k = 10400 ;     % spring constant
g = 9.81 ;      % gravity
b = 35 ;        % spring damping

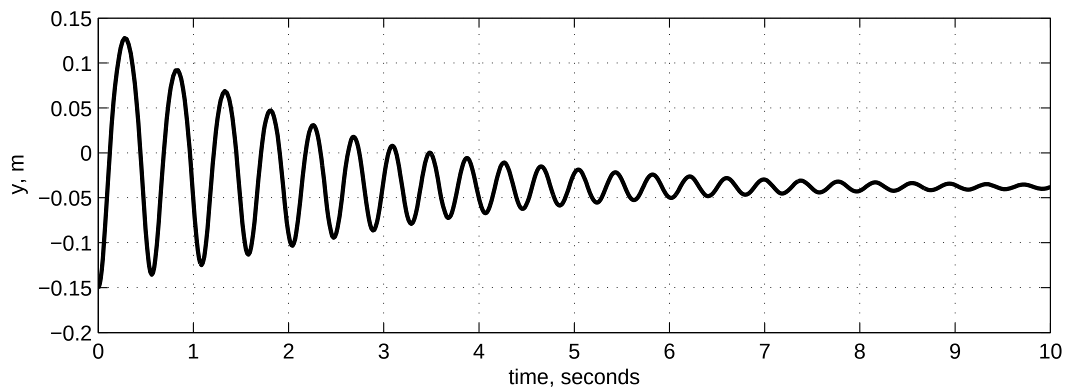
% first state is vertical velocity, second is position

if y(2) < 0,            % the spring is in action ...
    ydot(1,1) = 1/m * ( -k*y(2) - b*y(1)) - g ;
else,                   % ... or not
    ydot(1,1) = -g ;
end;

ydot(2,1) = y(1) ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



2
1
0
−1
−2
ydot, m/s
0
1
2
3
4
5
6
7
8
9
10



0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
y, m
0
1
2
3
4
5
6
7
8
9
10
time, seconds