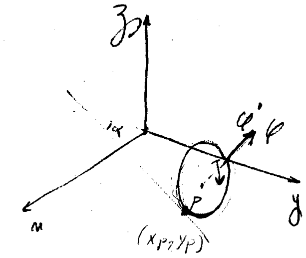
Session 14

Example 4

Vertical disk rotating on 2D plane



$\#\ DOF = 6 - 2 - 2 = 2$

$\downarrow$ αP

already accounted by the choice of $(x_p, y_p, \alpha, \varphi)$ $\leftarrow$ • $z_p = 0$ $v_x = \cdot$, $v_y = 0$

• nutation angle $= 0$

writing out the $v_P^{(m,y)} =$

$$0 = v_P^{(m,y)} = \underbrace{\dot x_p \hat i + \dot y_p \hat j}_{v_C} + \underbrace{(-\dot\varphi \cos\alpha \hat i + \dot\varphi \sin\alpha \hat j + \dot\alpha K)}_{\omega} \times (-R\underline{K})$$

$$\Rightarrow \begin{cases} \dot x_p - \dot\varphi R \sin\alpha = 0 \\ \dot y_p - \dot\varphi R \cos\alpha = 0 \end{cases}$$ non holonomic-Sclenomic

INTEGRABILITY OF CONSTRAINTS (digression)

Linear non holonomic Constraints

$$\sum_{i=1}^{N} c_{ji}(q,t)\dot q_i + b_j(q,t) = 0 \quad j = 1, \dots, m \quad (*)$$

Note that most Constraints in mechanics are linear in this sense Exception e.g.

Note: any holonomic Constraint $\{f_j(q,t) - d_j\}_{j=1,\dots m}$ can also be write in this form.

$$f_j(q_i(t), t) = 0 \Rightarrow \sum_{i=1}^{N} \frac{\partial f_j}{\partial q_i}\dot q_i + \frac{\partial f_j}{\partial t} = 0 \quad j = 1, \dots, m \quad (**)$$

Question: For (*), is there any $\{f_j\}_{j=1}^{m}$ such that (*) can be written in the form of (**)

Assume. Constraint is

$$\left.\begin{aligned} a_{ji} &= \frac{\partial f_j}{\partial q_i} \\ b_j &= \frac{\partial f_j}{\partial t} \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} \frac{\partial a_{ji}}{\partial q_k} &= \frac{\partial a_{jk}}{\partial q_i} \\ \frac{\partial a_{ji}}{\partial t} &= \frac{\partial b_j}{\partial q_i} \end{aligned}}$$ for any i, j, k.

Remaining issue (*) may become integrable after multiplication by integrating factor $c_j(q,t)$

$$\Rightarrow \sum \underbrace{(a_j c_j)}_{\tilde a_{ij}} \dot q_j + \underbrace{c_j b_j}_{\tilde \beta_j} = 0$$

$\Rightarrow$ $\underline{v}_P^{(xy)} = \underline{0}$ Constraint is truly non-holonomic and we can not, in obvious way, reduce the # of generalized coordinate

$\Rightarrow$ need to use l_1 coordinates in eq. of motion.

(3) Virtual Displacement
= infinitesimal displacements instantaneously compatible with the constraints

Notation: $\delta \underline{r}_i$; δq_k
Eq for a holonomic set of constraints

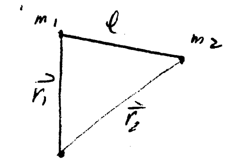
$$f_j(q_1, \dots, q_n, t) = 0 \quad j = 1, \dots, m$$

$$\sum_{l=1}^{N} \frac{\partial f_j}{\partial q_l} \delta q_l = 0 \quad j = 1, \dots, m$$

time is not varied

Example Dumbbell

$$(\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}_1 - \vec{r}_2) - \ell^2 = 0 \qquad \sum_{l=1}^{N} \frac{\partial f_j}{\partial q_l} \delta q_l = 0$$



For the virtual displacement

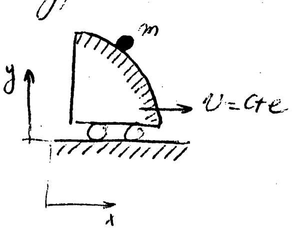
$$2(\underline{r}_1 - \underline{r}_2) \cdot (\delta \underline{r}_1 - \delta \underline{r}_2) = 0$$

$\Rightarrow$ $\delta r_1 - \delta r_2 \perp (r_1 - r_2)$
$\Rightarrow$ relative virtual displacement has no component in the direction of the beam.

$\Rightarrow$ Consistent with the physics: particles cannot have a relative displacement along the beam

Moving Circular track

(2) Moving Circular Track



Constrained
$$[x - v(t - t_0)]^2 + y^2 - R^2 = 0$$
rheonomic holonomic Constraint

$$2[x - v(t - t_0)]\delta x + 2y\, \delta y = 0$$

NOTE: Virtual displacement are not true infinitesimal displacement in rheonomic Syst