13.811
Advanced Structural Dynamics and Acoustics
Quiz - Acoustics
April 21, 2004

Question 1.

$$p_\omega(x,y;0) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ik_z h} p_\omega(k_x,k_y,h) e^{ik_x x} e^{ik_y y} dk_x dk_y \tag{4}$$

Solution Problem 1

Fourier transform of field at distance h

$$p_\omega(k_x,k_y;h) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_\omega(x,y,h) e^{-ik_x x} e^{-ik_y y} dx dy \tag{1}$$

Backpropagate to radiator

$$p_\omega(k_x,k_y;0) = p_\omega(k_x,k_y;h) e^{-ik_z h} \tag{2}$$

where k_z is the vertical wavenumber

$$k_z = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2}, & k_x^2 + k_y^2 \le k^2 \\ i\sqrt{k_x^2 + k_y^2 - k^2}, & k_x^2 + k_y^2 > k^2 \end{cases} \tag{3}$$

Question 2.

$$p_\omega(x,y,h) = \sin(\frac{\omega x}{2c}) = \frac{e^{ik_{x0}x} - e^{-ik_{x0}x}}{2i} \tag{5}$$

where $k_{x0} = k/2 = \omega/2c$. Insert into eq. 1 and use EGW eqn. 1.5

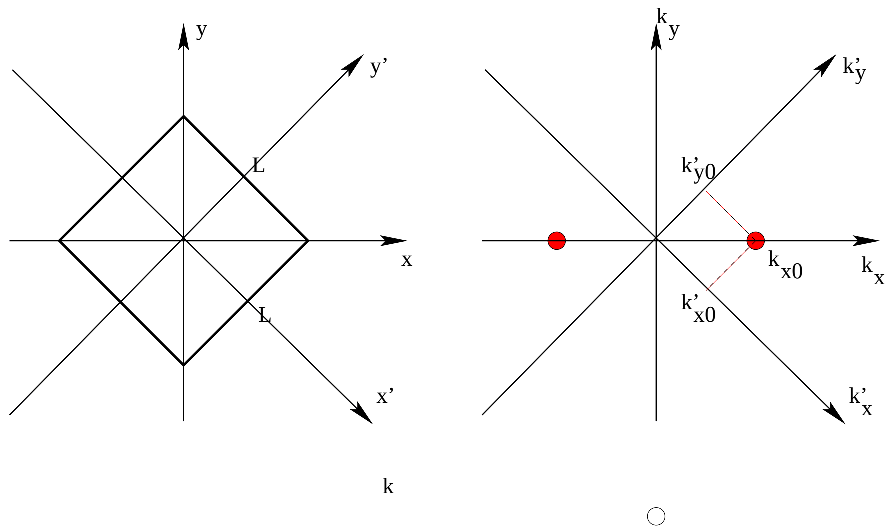
$$p_\omega(k_x,k_y;h) = \frac{4\pi^2}{2i}[\delta(k_x - k_{x0}) - \delta(k_x + k_{x0})]\delta(k_y) \tag{6}$$

Insert into Eq. (4)

$$\begin{aligned} p_\omega(x,y;0) &= \frac{1}{2i}[e^{-ik_{z0}h} e^{ik_{x0}x} - e^{-ik_{z0}h} e^{-ik_{x0}x}] \\ &= e^{-ik_{z0}h}\sin(\frac{\omega x}{2c}) \end{aligned} \tag{7}$$

with $k_{z0} = \sqrt{k^2 - k_{x0}^2} = k\sqrt{1 - 0.25} = k\sqrt{3}/2$

Solution Problem 2



$$\dot{w}(x,y,0) = \cos\frac{\omega x}{2c} = \frac{e^{ik_{x0}x} + e^{-ik_{x0}x}}{2} \tag{8}$$

with $k_{x0} = \omega/2c = k/2$. Wavenumbers in rotated coordinate system

$$k_x^{'} = \frac{k_x - k_y}{\sqrt{2}} \tag{9}$$

$$k_y^{'} = \frac{k_x + k_y}{\sqrt{2}} \tag{10}$$

$$\Rightarrow$$

$$k_{x0}^{'} = \frac{k_{x0}}{\sqrt{2}} = \frac{\omega}{2c\sqrt{2}} \tag{11}$$

$$k_{y0}^{'} = \frac{k_{x0}}{\sqrt{2}} = \frac{\omega}{2c\sqrt{2}} \tag{12}$$

Williams eq. 2.102 leads to

$$\begin{aligned} \dot{w}(k_x^{'}, k_y^{'}; 0) &= \frac{L^2}{2}[\mathrm{sinc}((k_x^{'} - k_{x0}^{'})L/2)\mathrm{sinc}((k_y^{'} - k_{y0}^{'})L/2) \\ &\quad + \mathrm{sinc}((k_x^{'} + k_{x0}^{'})L/2)\mathrm{sinc}((k_y^{'} + k_{y0}^{'})L/2)] \end{aligned} \tag{13}$$

Directivity function using transformations in eq. (9)-(12),

$$
\begin{aligned}
D(\theta, \phi) &= -\frac{i\rho\omega}{2\pi}\dot{w}(k_x, k_y, 0) \\
&= -\frac{i\rho\omega L^2}{4\pi}[\mathrm{sinc}((k_x - k_y - \omega/2c)L/\sqrt{2})\mathrm{sinc}((k_x + k_y - \omega/2c)L/\sqrt{2}) \\
&\quad +\mathrm{sinc}((k_x - k_y + \omega/2c)L/\sqrt{2})\mathrm{sinc}((k_x + k_y + \omega/2c)L/\sqrt{2})] \qquad (14)
\end{aligned}
$$

with

$$
\begin{aligned}
k_x &= k \sin\theta cos\phi \qquad (15) \\
k_y &= k \sin\theta sin\phi \qquad (16)
\end{aligned}
$$

Question 3.

Surface of radiater has a standing wavefield with crests parallel to the y-axis, generating two plane waves propagating at grazing angle $\cos^{-1}(1/2) = 60°$, interfering at all distances h to produce a standing wavefield in the horizontal plane, but propagating vertically