LINEARIZED ENERGY-STORING TRANSDUCER MODELS

Energy transduction in an electro-mechanical solenoid may be modeled by an energy-storing multiport.

$$\xrightarrow[\;i\;]{\;e = \dot\lambda\;} \text{IC} \xleftarrow[\;\dot{x}\;]{\;F\;}$$

Energy transduction in an electric motor may be modeled by a gyrator.

$$\xrightarrow[\;i\;]{\;e = \dot\lambda\;} \text{G Y} \xrightarrow[\;\dot{x}\;]{\;F\;}$$

But the fundamentals of energy transduction are the same in both.

How can these different models be reconciled?

Linearizing the nonlinear multiport constitutive equations reveals their relation.

LINEARIZED SOLENOID MODEL

A gyrator relates power variables (effort and flow)

A multiport energy storage element relates power and energy variables (effort and momentum, flow and displacement)

Solenoid model constitutive equations:

$$\lambda = i\, L\, e^{-(x/x_c)2}$$

$$F = \frac{i2\, L\, x\, e^{-(x/x_c)2}}{x_c2}$$

Differentiate the electrical constitutive equation with respect to time.

$$e = \frac{d\lambda}{dt}$$

$$\lambda = i\, L\, e^{-(x/x_c)2}$$

$$\frac{d\lambda}{dt} = \left(L\, e^{-(x/x_c)2}\right)\frac{di}{dt} - \left(\frac{2\, i\, L\, x\, e^{-(x/x_c)2}}{x_c2}\right)\frac{dx}{dt}$$

Consider small deviations about an operating point:

nominal position x_O

nominal current i_O

Define
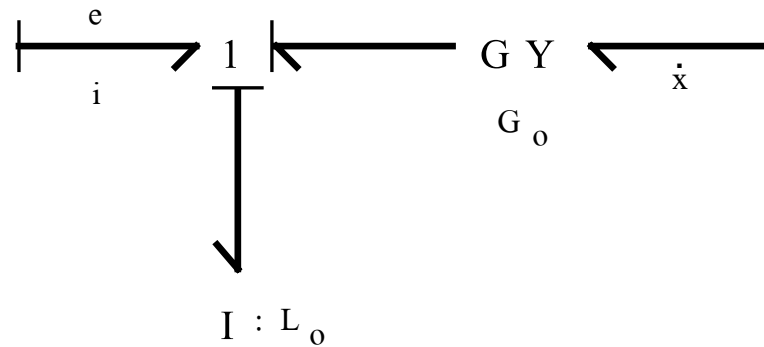
$$L_O \triangleq L\, e^{-(x_O/x_C)^2}$$

and

$$G_O \triangleq \frac{2\, i_O\, L\, x_O\, e^{-(x_O/x_C)^2}}{x_C^2}$$

Linearized relation

$$e = L_O \, di/dt - G_O \, dx/dt$$

This equation describes an inductor and gyrator connected as shown in the following bond graph fragment.

The inductor has differential causality because the solenoid model has differential causality on the electrical port.

Differentiate the mechanical constitutive equation with respect to time.

$$\frac{dF}{dt} = \left(\frac{2\,i\,L\,x\,e^{-(x/x_c)^2}}{x_c^2} \right) \frac{di}{dt}$$

$$+ \left(\frac{(1 - 2(x/x_c)^2)\,L\,i^2\,e^{-(x/x_c)^2}}{x_c^2} \right) \frac{dx}{dt}$$
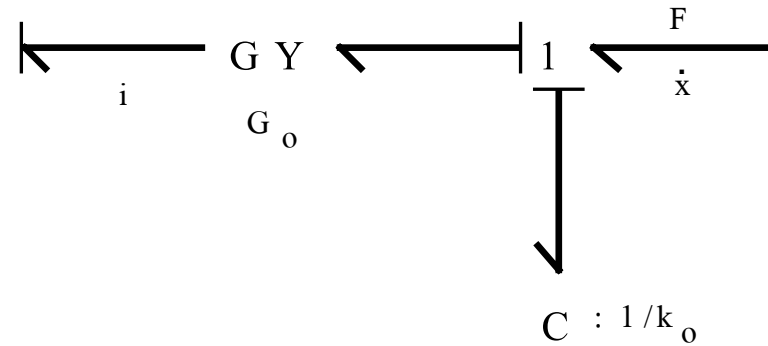
Assuming the same operating point:
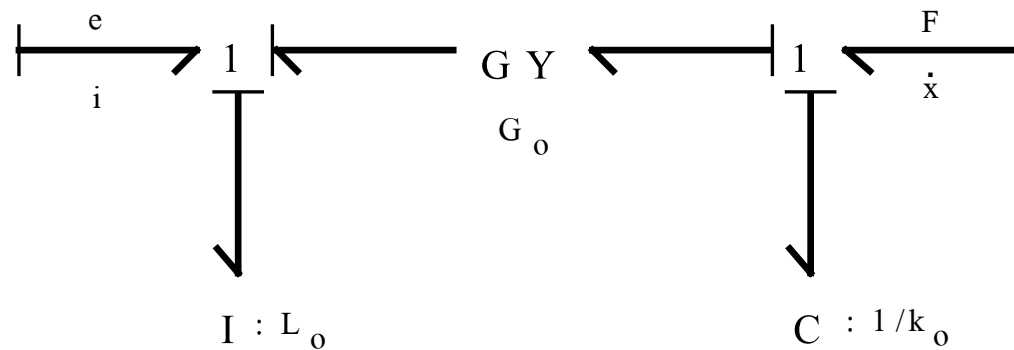
$$dF/dt = G_o\,di/dt + k_o\,dx/dt$$

where

$$k_o \triangleq \frac{(1 - 2(x_o/x_c)^2)\,L\,i^2\,e^{-(x_o/x_c)^2}}{x_c^2}$$

This equation describes a spring and gyrator connected as shown in the following bond graph fragment.

$$GY \qquad 1 \qquad F \qquad \dot{x}$$

(bond graph fragment)

$\xleftarrow{\quad}$ G Y $\xleftarrow{\quad}$ 1 $\xleftarrow{F}$

i G_o $\dot{x}$

$\downarrow$

$C \; : \; 1/k_o$

Combine the bond graph fragments for the two linearized constitutive equations.

Linearized multiport bond graph:

Similar to our model of electromagnetic transduction in a DC permanent magnet motor.

SOLENOID MODEL VS. DC MOTOR MODEL

The linearized multiport solenoid model is a combination of ideal gyrator, ideal inertia and ideal capacitor

From the electrical port the multiport IC "looks like" an inertia (inductor)

    —hence the inertia (inductor)

From the mechanical port the multiport IC "looks like" a capacitor (spring)

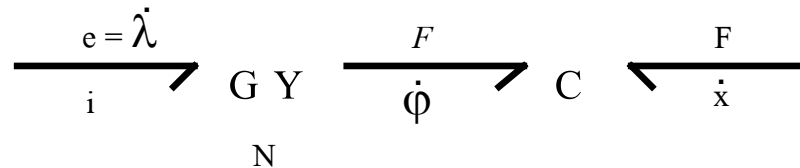   **—hence the capacitor (spring)**

WHY THE GYRATOR?

the multiport IC contains a "hidden" gyrator.

A more detailed solenoid model has

a gyrator for the electromagnet

a multiport capacitor for the magnetic field.

$$\xrightarrow[\quad i \quad]{\quad e = \dot\lambda \quad} \;\; GY \;\; \xrightarrow[\quad \dot\phi \quad]{\quad F \quad} \;\; C \;\; \xleftarrow[\quad \dot x \quad]{\quad F \quad}$$

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WHY BOTHER WITH THE MULTIPORT MODEL?

   — the inertia-gyrator-capacitor model is arguably simpler.

THE MULTIPORT MODEL OFFERS DEEPER INSIGHT

   — and that's the main purpose of modeling.

   Nonlinear behavior is common in electro-magnetic transduction.

   The inertia-gyrator-capacitor model is a *linearized approximation.*

A typical DC permanent-magnet motor is cunningly designed with many coils and a commutation system (mechanical or electrical) that switches current from one coil to another as the motor rotates so that each coil operates in a range where its behavior is close to linear under most conditions.

   The inertia-gyrator-capacitor model is one *decomposition* of the multiport IC.

   Other multiport decompositions may be defined.

THE MULTIPORT MODEL SHOWS HOW THE DEVICE MAY DEVIATE FROM IDEAL GYRATOR BEHAVIOR.

A "parasitic" inductance may be expected because a solenoid (or motor) operates by creating a magnetic field.

It is probably less obvious that a "parasitic" spring may be expected.

THE MULTIPORT MODEL SHOWS THAT FORCE (OR TORQUE) DEPENDS ON ARMATURE DISPLACEMENT (IN GENERAL).

In general, any electromagnetic transducer may exhibit "spring-like" behavior.

In special circumstances, the apparent stiffness may be zero

— e.g., $k_0 = 0$ if $x_0 = x_c/\sqrt{2}$

In fact, despite cunning design, position-dependent force (torque) is one of the more noticeable forms of non-ideal DC motor behavior — usually termed "cogging torque".

REMARK:

Any transduction device may serve as an actuator or as a sensor. A solenoid is one of the commonest linear actuators. An LVDT (linear variable differential transformer) uses a displacement-modulated inductance change to measure position.