13.49 Homework #5

1. You are asked to assess the operational envelope of a cable/vehicle system which has been installed on a vessel. The cable is steel-jacketed, with diameter $3cm$, an effective extensibility of $100 \times 10^9 Pa$, and a density of $5000kg/m$. The vehicle is streamlined, and has a mass (material plus added) of $100kg$.

   (a) The towing angle at the vessel cannot exceed $25deg$ from vertical, for reasons of deck and crane geometry. If this angle is considered equal to the critical angle ϕ_c, what is the maximum speed for the system? Assume the normal drag coefficient of the cable is $C_n = 1.2$.

   (b) Considering the undamped, uncoupled axial dynamics, express the natural frequency as a function of the vehicle depth, and sketch the curve. If the fastest waves that the surface vessel responds to have period $4s$, what is the "threshold" operating depth?

   (c) The undamped, uncoupled lateral dynamics follow the equation

$$(m + m_a)\frac{\partial^2 q}{\partial t^2} = \bar{T}\frac{\partial^2 q}{\partial s} + w_n \sin\bar{\phi}\frac{\partial q}{\partial s},$$

   where m_a is the added mass of the cable per unit length, and w_n is the in-water weight of the cable. Separation of variables $q(s,t) = \tilde{q}(s)\cos\omega t$ gives

$$\bar{T}\frac{\partial^2 \tilde{q}}{\partial s^2} + w_n \sin\bar{\phi}\frac{\partial \tilde{q}}{\partial s} + w_n \gamma \tilde{q} = 0,$$

   where $\gamma = (m + m_a)\omega^2/w_n$. Simplify this expression by writing $\bar{T} = w_n s$ (the vehicle weight effect is small), $\sin\bar{\phi} \simeq 1$ (the cable is nearly vertical), and then use the substitution $z = 2\sqrt{\gamma s}$ to arrive at the Bessel equation

$$z^2\frac{\partial^2 \tilde{q}}{\partial z^2} + z\frac{\partial \tilde{q}}{\partial z} + z^2\tilde{q} = 0.$$

   The derivatives can be transformed from s to z coordinates via the chain rule:

$$\begin{aligned}\frac{\partial \tilde{q}}{ds} &= \frac{\partial \tilde{q}}{\partial z}\cdot\frac{\partial z}{\partial s}\\ \frac{\partial^2 \tilde{q}}{ds^2} &= \frac{\partial^2 \tilde{q}}{\partial z^2}\left(\frac{\partial z}{\partial s}\right)^2 + \frac{\partial \tilde{q}}{\partial z}\cdot\frac{\partial^2 z}{\partial z \partial s}\cdot\frac{\partial z}{\partial s}.\end{aligned}$$

   (d) The equation above has a solution of the form $\tilde{q} = cJ_0(z)$, where c is a constant to be found, and $J_0(z)$ is the zero'th order Bessel function of the first kind. If $\tilde{q}(s = L) = Q$, that is, we impose a harmonic input at the top, find c in terms of Q and $z(s = L)$. What condition makes $\tilde{q}$ blow up, indicating a resonance frequency ω_n?

   (e) Recalling that strumming occurs when $\omega_n \simeq 0.2U/d$, construct a graph of towing velocities vs. deployment depths for which the strumming would be centered. Include at least the three lowest modes in your sketch, and discuss the trends for the higher modes. $J_0(x) = 0$ for for $x \simeq \pi(n - 0.25), n = 0, 1, 2, \cdots$, especially when x is large. You should find that strumming is hard to avoid in deep towing applications!

2. The coefficients govering the pitch/heave dynamics of a Deep Submergence Rescue Vehicle (DSRV) are given below. They are nondimensionalized as in the lecture notes, using the factors $\rho/2$, U, and L.

$I'_{xx} = 0.000118$	$M'_q = -0.0113$	$Z'_q = -0.0175$
$I'_{zz} = 0.00193$	$M'_{\dot{q}} = -0.00157$	$Z'_{\dot{q}} = -0.000130$
$x'_G = 0$	$M'_w = 0.0112$	$Z'_w = -0.0439$
$m' = 0.0364$	$M'_{\dot{w}} = -0.000146$	$Z'_{\dot{w}} = -0.0315$
$U = 2.0 m/s$	$M'_\delta = -0.0128$	$Z'_\delta = -0.0277$
$L = 15.0 m$	$M'_\theta = -0.156/U^2$	

(a) Write the linearized (nondimensional) dynamics in the matrix form $\dot{x} = Ax + Bu$, where the input $u = \delta$, and the state vector is $x = [w', q', \theta, Z']$. Z' here is the elevation of the vehicle in inertial coordinates; your approximation for $\dot{Z}'$ should include both w' and θ.

(b) Is the DSRV vehicle open-loop stable, that is, without control action δ? You can assess this either by finding the eigenvalues of your A-matrix above, or by computing the stability parameter C'.

(c) In preparation for controller design, create a simulation using Matlab, as in Homework 1. Make a graph of the open-loop step response, showing all the dimensional state variables versus dimensional time.

(d) Assuming that all the states can be measured accurately, a full-state controller $\delta = -Kx$ can be used, where K is a 1×4 gain matrix. Under this control, the system dynamics are governed by $\dot{x} = (A - BK)x$, so that if $A - BK$ has all negative eigenvalues, the system is stable.

Use the Matlab function *place* to put the four closed-loop poles at $g(-0.95 \pm 0.31i)$ and $g(-0.59 \pm 0.81i)$, $g = 1$. Demonstrate that your closed-loop design is stable against nonzero initial conditions. What are the effects of increasing or decreasing g?

Note that the poles locations suggested above are with respect to the *dimensional* system, i.e., using dimensional time.