Department of Mechanical Engineering
Massachusetts Institute of Technology

2.160 Identification, Estimation, and Learning

End-of-Term Examination

May 17, 2006

1:00 – 3:00 pm (12:30 – 2:30 pm)

Close book. Two sheets of notes are allowed.
Show how you arrived at your answer.

Problem 1 (40 points)

Consider the neural network shown below. All the units are numbered 1 through 6, where units 1 and 2 are input units relaying the two inputs, x_1 and x_2, to units 3 and 4. The output function of each unit is a sigmoid function;

$$y_i = g_i(z_i) = \frac{1}{1+e^{-z_i}},$$

where variable z_i is the weighted sum of all the inputs connected to that unit:

$$z_i = \sum_j w_{ij} x_j \; .$$

There are three hidden units, units 3, 4 and 5, connected to the output unit, unit 6.

The output of the network, y_6, is connected to a known, but nonlinear process:

$$\hat{y} = y_6^{\;3}$$

A distal teacher provides a training signal *y*, which is compared to the above estimate $\hat{y}$. The weights of the network are corrected based on the error back propagation algorithm with learning rate η to reduce the squared error given by:

$$E = \frac{1}{2}(\hat{y} - y)^2 .$$

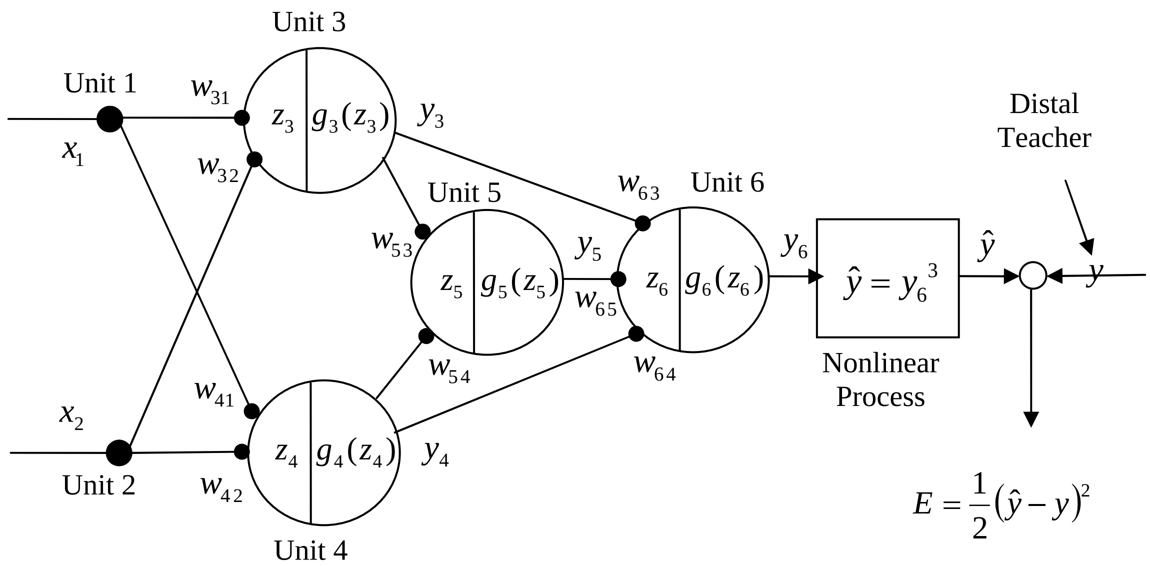
Answer the following questions.

a). Using the chain rule, compute the incremental weight change to be made to w_{54} when inputs x_1 and x_2 and the corresponding distal teacher signal *y* are presented.

b). Compute the incremental weight change Δw_{41} for the same input-output training data as part a).

c). In computing Δw_{41}, discuss how the result of computing Δw_{64} as well as that of Δw_{54} can be used for streamlining the computation.

d). For a particular pair of input data, z_6 became very large, $z_6 \gg 1$. Is the weight change Δw_{31} large for this input? For another input, z_5 became very large $z_5 \gg 1$, and z_6 and z_3 became 0. Are weight change Δw_{31} and Δw_{53} likely to be large when *E* is large? Explain why.



Problem 2 (60 points)

Consider the following true system and model structure with parameter vector θ,

$$S: \quad y(t) + 0.3y(t-1) + 0.1y(t-2) = u(t-1) + e_0(t)$$
$$M(\theta): \; y(t) + a_1 y(t-1) + a_2 y(t-2) = u(t-1) + e(t) + ce(t-1)$$
$$\theta = (a_1, a_2, c)^T$$

where input sequence $\{u(t)\}$ is white noise with variance μ and $\{e_0(t)\}$ is white noise with variance λ. The input $\{u(t)\}$ is uncorrelated with noise $\{e_0(t)\}$ and $\{e(t)\}$. Answer the following questions.

1). Obtain the one-step-ahead predictor of $y(t)$.

2). Compute covariances

$$R_{ye}(0) = E[y(t)e_0(t)], \; and \; R_{ye}(1) = E[y(t)e_0(t-1)].$$

3). Compute covariances

$$R_y(0) = E[y^2(t)], \; R_y(1) = E[y(t)y(t-1)]$$

4). Obtain the asymptotic variance of parameter estimate: $\hat{\theta}_N = [\hat{a}_1, \hat{a}_2, \hat{c}]^T$, when a quadratic prediction-error criterion is used.

5). After identifying the parameters $\hat{a}_1$ and $\hat{a}_2$, the true system has changed to:

$$S: \quad y(t) + 0.3y(t-1) + 0.1y(t-2) = u(t-1) + e_0(t) + 0.2e_0(t-1).$$

Now consider the model structure:

$$M(\theta): \; y(t) + 0.3y(t-1) + 0.1y(t-2) = u(t-1) + e(t) + ce(t-1)$$

with only one unknown parameter $\theta = (\, c\,)$. Obtain the asymptotic error covariance using the frequency-domain expression of Cov $\hat{\theta}_N$. [Hint, obtain the function $A(\omega)$ involved in the following expression:

$$Cov\hat{\theta}_N \sim \frac{1}{N}\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} A(\omega)\, d\omega\right]^{-1}$$

using the parameter values of the true system.]

Problem 3 (for extra points)

What are the two most important or most inspiring things that you have learned in 2.160 that you think you should not forget even ten years after your graduation from MIT?