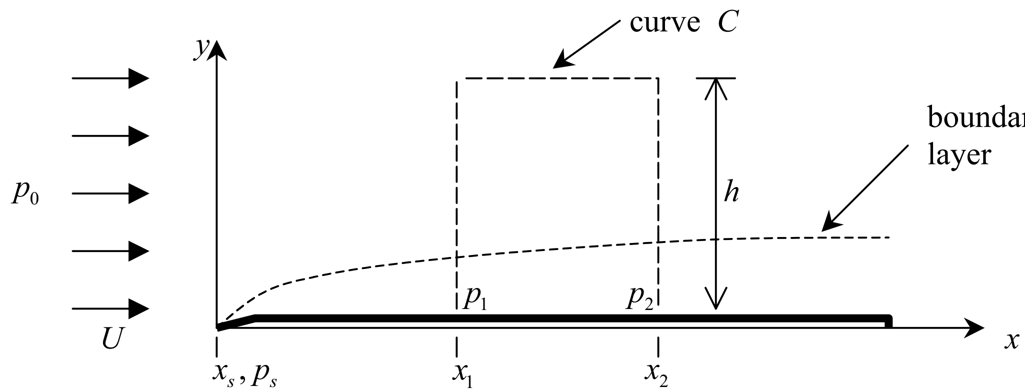
Challenge Problem 3 (OPTIONAL) **Name: ____________________**

1. Consider a real fluid ($\nu \neq 0$) of constant density. Use the Navier-Stokes Equations to show that

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\oint_C \vec{V}\cdot d\vec{l} = -\iint_A \nabla\times(\vec{\omega}\times\vec{V})\cdot\hat{n}dS + \oint_C \nu\nabla^2\vec{V}\cdot d\vec{l} \qquad (1)$$

where *C* is a closed curve fixed in space and *A* is any surface bounded by *C*.
Hint: Use the identity $(\vec{V}\cdot\nabla)\vec{V} = \vec{\omega}\times\vec{V} + \nabla(\frac{1}{2}\vec{V}\cdot\vec{V})$.

2. A real, constant density fluid flows over a thin flat plate placed in a free stream of velocity $\vec{V} = U\hat{i}$, creating a boundary layer. Assume that the flow is steady. Consider a rectangular curve fixed with respect to the plate and boundary layer as shown:



The point x_s at the leading edge of the plate is a stagnation point where the stagnation pressure is p_s. The pressure at point x_1 is p_1. Point x_2 is assumed to be far enough downstream in the boundary layer that p_2 equals the free stream pressure p_0.

(a) Use equation (1) in Problem 1 to show that

$$\frac{p_2}{p_1} = \int_0^h (u\zeta)_{x=x_2}dy - \int_0^h (u\zeta)_{x=x_1}dy \qquad (2)$$

where $\vec{V} = u\hat{i} + v\hat{j}$ and $\vec{\omega} = \zeta\hat{k}$ for two-dimensional flow.

Hint: Evaluate the Navier-Stokes equations at $y = 0$ on the plate to obtain a relationship between the pressure gradient and velocity gradient. Use this relation to evaluate the line integral.

Note that the right-hand side term in equation (2) represents the *net* flux of vorticity transported or convected in the horizontal direction through the vertical sections at x_1 and x_2 by the velocity component *u*.

(b) Now move the left side of the rectangular curve *C* to the leading edge of the plate so that $x_1 = x_s$, and leave x_2 fixed.

(i) Show that equation (2) becomes

$$\frac{-U^2}{2} = \int_0^h (u\zeta)_{x=x_2} dy \tag{3}$$

(ii) If x_2 is *any* point in the region of the boundary layer where $p_2 = p_0$ (in other words, where $dp/dx = 0$), what can you say about the convection of vorticity through any vertical section in this region?

(c) Now imagine that x_1 is downstream of the leading edge and near x_2, so that $p_1 = p_2 = p_0$.

(i) Write equation (2) for this case.

(ii) What is the *net* convection of vorticity through any two vertical sections in the region of the boundary layer where $dp/dx = 0$?

(d) Based on your answers to (b) and (c), what can you conclude about *where* vorticity is introduced into a flat-plate boundary layer?