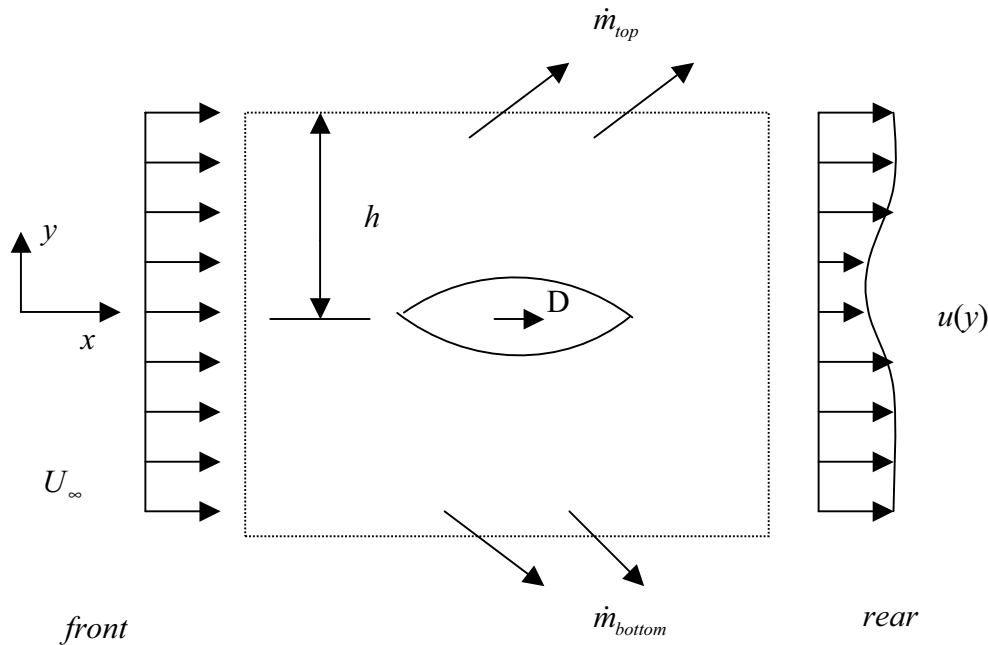1. Consider an airfoil where the flow over the foil is two-dimensional (this is the same as assuming the foil is infinitely long in the z-direction, out of the page). We want to find the drag force D in the x-direction on a section of the foil having width w in the z-direction. Assume the following for the figure shown:

- The velocity profile at a downstream location is known and is $u(y)$

- The flow is *steady* and symmetric around the foil (the foil is not inclined to the flow)

- The front and rear surfaces of the control volume shown is chosen to be at large distances up and downstream from the foil so that the pressure is atmospheric

- The value of h is chosen large enough so that the x-component of the air velocity $\bar{V} = (u, v)$ at the top and bottom surfaces of the control volume is *approximately* the same as the "free-stream" velocity U_∞.

(a) First, use the conservation of momentum for a fixed control volume to write an expression for the total force on the fluid in the control volume in terms of surface integrals over the front, rear, top and bottom surfaces of the control volume. Note that by assumption of 2-D flow, there is no contribution to the momentum flux from the two end

surfaces of the control volume that have normals in the *z*-direction. Your final answer should be in terms of the drag *D*, foil width *w*, constant density ρ, velocity components U_∞ and *u(y)*, height *h*, and the mass fluxes through the top and bottom surfaces, $\dot{m}_{top}$ and $\dot{m}_{bottom}$. The momentum fluxes for the front and rear surfaces will be left as integrals, while the momentum fluxes for the top and bottom can be written without integrals in terms of $\dot{m}_{top}$ and $\dot{m}_{bottom}$. Note that by symmetry, $\dot{m}_{top} = \dot{m}_{bottom}$.

Suggestion: Make use of symmetry to write two of the integrals with limits from *y=0* to *y=h*.

(b) Based on the inlet and exit velocity profiles, how do you know $\dot{m}_{top} = \dot{m}_{bottom} \neq 0$?

(c) Now use the conservation of mass for the same control volume to solve for $\dot{m}_{top}$ $(= \dot{m}_{bottom})$ as a single integral in terms of the foil width *w*, density ρ, velocity components U_∞ and *u(y)*, and height *h*.

(d) Finally, substitute the expression for $\dot{m}_{top}$ obtained in (c) into the momentum equation obtained in (a). Solve for the drag *D* acting ON the foil as a single integral in terms of the foil width *w*, density ρ, velocity components U_∞ and *u(y)*, and height *h*.

(e) Assume that a crude approximation to the downstream symmetrical velocity profile in the wake of the foil is the linear profile $u(y) = 20|y| + 300$ feet per second (fps), the free-stream velocity $U\infty = 500$ fps, and the density is 0.0024 slugs per cubic foot. Find the drag on a 50-foot wide section of the foil.

(f) For part (e), find the power (in horsepower) needed to propel the section of foil at 500 fps.

2. Complete the following questions:

(a) To prove that the stress tensor is symmetric, we make use of the conservation of _____ momentum and conservation of _____ momentum for a small fluid element.

(b) Pascal's Law states that in a fluid at rest, the pressure acting normal to any plane surface passing through a point (does not change, depends on the orientation of the surface).

(c) Euler's equations are the differential form of the conservation of _____ .

(d) The "dynamic" stress $\hat{\tau}_{ij}$ is proportional to the velocity _____ in a Newtonian fluid. The constant of proportionality for an incompressible, isotropic, Newtonian fluid is called the coefficient of _____ viscosity.

(e) The quantity $\partial \tau_{ij} / \partial x_j$ has dimensions of

- Stress
- Force per unit volume
- Acceleration
- None of the above

(f) The quantity $\nu \dfrac{\partial^2 u_i}{\partial x_j \partial x_j}$, where ν is the kinematic viscosity, has the dimensions of

- Stress
- Acceleration
- Force
- None of the above

(g) The four unknowns in the Navier-Stokes and continuity equations are: