13.42 Homework #3

Spring 2005

Out: Thursday, February 17, 2005

Due: Thursday, February 24, 2005

<u>Problem 1:</u> *Probability Review Problem.* Two fair dice are rolled simultaneously with the following defined events:

A = {Sum of roll is even}

B = {Sum of roll is odd}

C = {Doubles are rolled}

D = {Sum of roll is ≤ 8 }

Find the following probabilities:

a) $P(A \cup D)$

b) $P(B \cap C)$

c) $P(C \cup D)$

d) $P(B \cap D)$

e) $P(A \cap C)$

<u>Problem 2:</u> *Probability Review Problem.* Find the probability of drawing a full house in a five-card hand. (Assume that every possible five-card hand drawn from a standard deck of 52 cards has the same probability of being selected.)

<u>Problem 3:</u> Let random variable X be the total sum of the outcomes of 3 flips of a fair coin when the value of 1 is assigned to heads and the value of 0 is assigned to tails.

a) Find the probability mass function (i.e. the discrete version of the probability density function), $p_X(x) = P(X = x)$.

b) Find the cumulative distribution function, $F_X(x) = P(X \leq x)$.

c) Using the probability function, find the mean, variance, and standard deviation.

Problem 4: Let the random variable X have a cumulative distribution function $F_X(x)$:

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4}x + \frac{1}{4} & \text{if } -1 \le x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < \frac{1}{2} \\ x^2 & \text{if } \frac{1}{2} \le x < 1 \\ 1 & \text{if } 1 \le x \end{cases}$$

Find the following probabilities:

a) $P(x \le \frac{1}{2})$

b) $P(x \ge 0)$

c) $P(0 \le x < \frac{1}{2})$

d) $P(x \le \frac{3}{4})$

e) $P(x \ge 1)$

Problem 5: Let Y be a random variable with the following probability density function:

$$f_X(x) = \begin{cases} 2x & \text{if } 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the expected value, μ_X .

b) Find the standard deviation, σ_X .

Problem 6: Consider a random process, wave elevation:

$$\eta(t) = A\cos(\omega t + \phi)$$

where ω and ϕ are constants and A is a Gaussian random variable with zero mean.

a) What is the probability density function of A?

b) What is the dynamic pressure, $p_d(t)$, under this wave elevation?

c) Determine whether $p_d(t)$ is a stationary process.

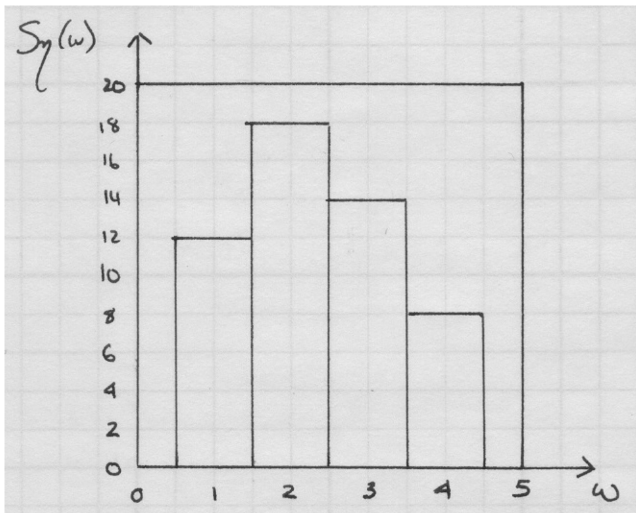
Problem 7: (MATLAB recommended for this problem.)

The random wave elevation at a given point in the ocean may be represented as follows:

$$\eta(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i)$$

Where A_i, ω_i, *and* ϕ_i are the wave amplitude, frequency, and random phase angle (with uniform distribution from 0 to 2π) of wave component number *i*.

The amplitudes A_i corresponding to each frequency ω_i may be found from a known wave energy spectrum, $S_\eta(\omega)$, where $S_\eta(\omega)\delta\omega = \frac{1}{2}A_i^2$.



a) Using the wave energy spectrum given in Figure 1, generate 10 realizations of the random process $\eta(t)$. Let t = [0, 90] seconds. (Hint: Use the MATLAB function *rand* to generate a realization of the random variable ϕ_i .)

b) Compute the ensemble averages (mean and variance at t = 30 seconds, and compute the temporal averages (mean and variance) for each realization. Compare the results.

Problem 8: Give short answers to the following questions.

a) What causes the vast majority of sea waves?
b) Why do storm waves have a limiting wavelength?
c) What is significant wave height?
d) Why is significant wave height used so extensively in design practices?