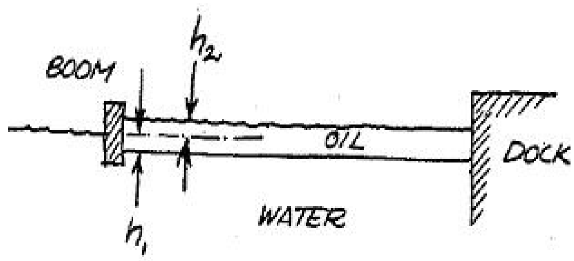
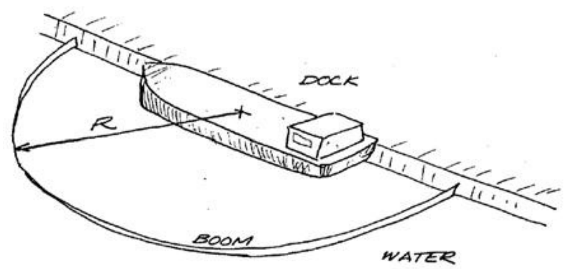
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 1.03

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin





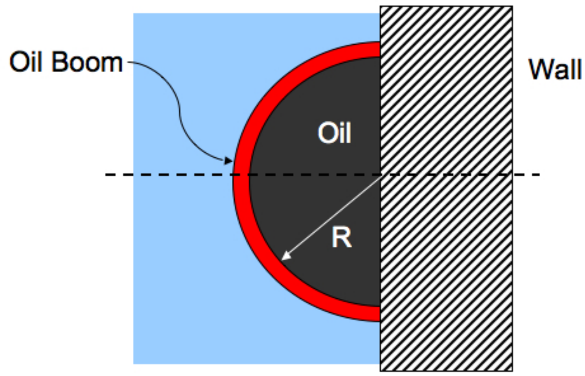
Oil Spills may occur in ports where oil tankers are loaded. The density of oil, ρ_o is less than that of water ρ_w, and the two fluids are immiscible, so that when a spill occurs the oil simply spreads out in a layer on top of the water. To contain any possible spills, a semi-circular "oil boom" is deployed at a radius R around the dock where the loading takes place.

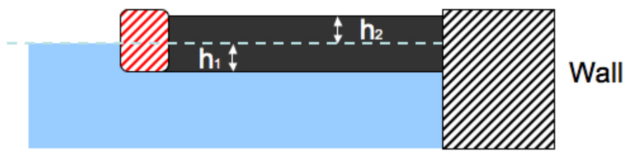
The boom is a barrier which floats on the water, its bottom submerged and its top a bit above the water surface, as shown. This barrier prevents the oil from spreading past it, at least if the spill is not too great (see part b).

Suppose a volume V of oil is spilled inside the boom. After sufficient time has elapsed for the situation to reach static conditions, calculate, in terms of ρ_o, ρ_w, R, V and g,

- (a) the depth h_1 of the bottom surface and the elevation h_2 of the top surface of the contained oil relative to the water surface outside the boom;
- (b) the components of force parallel to and transverse to the dock exerted by one of the moored boom ends on the dock.

Solution:





(a) The total volume of oil is:

$$V = \frac{\pi R^2}{2}(h_1 + h_2) \tag{1.03a}$$

If atmospheric pressure, P_a, is constant above the oil and water, then $P = P_a + \rho_o g h$ in the oil; and $P = P_a + \rho_w g h$ in the water, where h is the depth measured form the surface of each fluid. In the oil, just above the oil/water interface,

$$P_{interface} = \rho_o g(h_1 + h_2) + P_a \tag{1.03b}$$

In the water, just below the interface,

$$P_{interface} = \rho_w g h_1 + P_a \tag{1.03c}$$

Since the pressure must be continuous across the interface,

$$\rho_o g(h_1 + h_2) + P_a = \rho_w g h_1 + P_a \tag{1.03d}$$

Solving (1.03a) and (1.03d) for h_1 and h_2 we obtain

$$h_1 = \frac{2V}{\pi R^2}\frac{\rho_o}{\rho_w} \tag{1.03e}$$

$$h_2 = \frac{2V}{\pi R^2}\frac{\rho_w - \rho_o}{\rho_w} \tag{1.03f}$$

(b) Lets consider forces on the bottom arising from the pressure changes in the oil and water. The net outward pressure force per unit circumference of the bottom, F/length is

$$\frac{F}{length} = \int_0^{h_1+h_2} P_{oil} dh - \int_0^{h_1} P_{water} dh - \int_0^{h_2} P_a dh \tag{1.03g}$$

$$\frac{F}{length} = \frac{\rho_o g(h_1+h_2)^2}{2} - \frac{\rho_w g h_1^2}{2} \tag{1.03h}$$

$$\frac{F}{length} = = \frac{\rho_o g}{2}\left(\frac{2V}{\pi R^2}\right)^2 \frac{\rho_w - \rho_o}{\rho_w} \tag{1.03i}$$

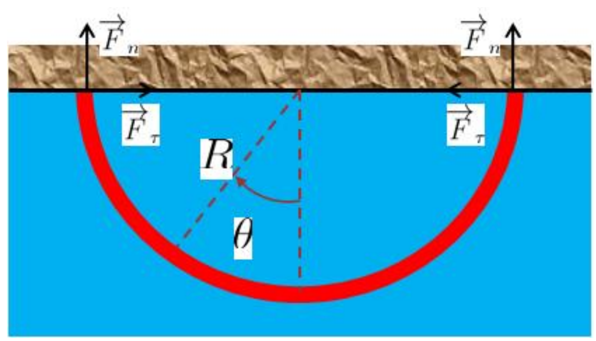
(Notice that the Atmospheric Pressure Terms Cancel Out)

Now, for the transverse (to the dock) component, consider a small element of boom of length $Rd\theta$. The net outward radial force per length on this element is $(F/length)Rd\theta$, and thus the net normal component of this force is:

$$F_{net,n} = 2\int_0^{\pi/2} \left(\frac{F}{length}\right) cos\theta R d\theta = 2\left(\frac{F}{length}R\right) \tag{1.03j}$$

Notice that the integral obtained before is just the projection of the circumference length over the dock, and the result is simply the boom diameter. Since the bottom is in static equilibrium, the above force is also applied to the bottom by the dock at the two moorings. Hence, the force exerted by one of the moorings on the dock, F_{M_n}, is actually half of the above and in the opposite direction,

$$F_{M_n} = -0.5F_{net,n} = \frac{\rho_o g}{2R^3}\left(\frac{2V}{\pi}\right)^2 \frac{\rho_w - \rho_o}{\rho_w} \tag{1.03k}$$



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Problem Solution by MK/MC(updated)/JK(updated), Fall 2009

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