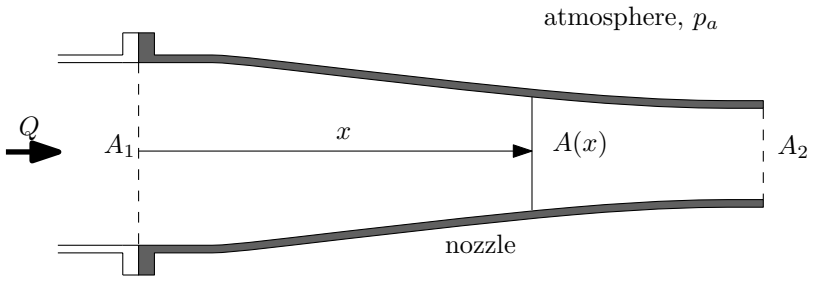
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 4.04
This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A nozzle with exit area A_2 is mounted at the end of a pipe of area A_1, as shown. The nozzle converges gradually, and we assum that the flow in it is (i) approximately uniform over any particular station x, (ii) incompressible, and (iii) inviscid. Gravitational effects are, furthermore, taken as negligible. The volume flow rate in the nozzle is given as Q and the ambient pressure is p_a.

(a) Derive an expression for the gage pressure at a station where the area is $A(x)$.

(b) Show, by integrating the x-component of the pressure force on the nozzle's interior walls, that the net x-component of force on the nozzle due to the flow is independent of the specific nozzle contour and is given by

$$F = \rho Q^2 \frac{(A_1 - A_2)^2}{2A_1 A_2{}^2}$$

(c) The expression in (b) predicts that F is in the positive x-direction regardless of whether the nozzle is converging ($A_2 < A_1$) or diverging ($A_2 > A_1$). Explain.

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