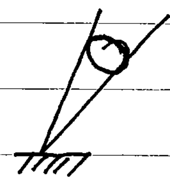
1. Summary of Lecture 1

Planck's Law	emissive power
Wien's displacement law	intensity
Stefan-Boltzmann law	solid angle.

2. Correction to intensity



$$I_\lambda' = \frac{P_\lambda'}{dA_\perp d\lambda d\Omega} = \frac{P_\lambda'}{dA\cos\theta\, d\lambda d\Omega}$$

$$\cancel{E_\lambda = \frac{\int P_\lambda' d\Omega}{dA}}$$

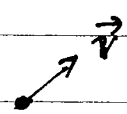
$$P_\lambda = I_\lambda \cos\theta\, dA\, d\lambda\, d\Omega$$

$$P_\lambda = \int_\Omega I_\lambda \cos\theta\, dA\, d\lambda\, d\Omega$$

$$E_\lambda = \frac{P_\lambda}{d\lambda dA} = \int_\Omega I_\lambda \cos\theta\, d\Omega .$$

3. Comments.

a. I is a scalar



(x, y, z), (v_x, v_y, v_z) $\Rightarrow$ distribution function

$f(x, y, z, v_x, v_y, v_z, t)$

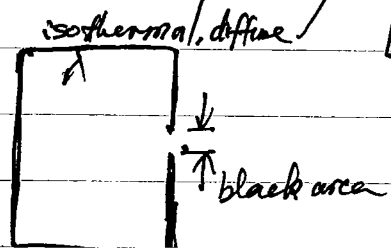
(phase space

b. How to make a blackbody

blackbody also best absorber

~~Hohlra~~

Hohlraum



Lecture 2. 2/1

$$dA_p = R^2 \sin\theta \, d\theta \, d\varphi$$

$$d\Omega = \sin\theta \, d\theta \, d\varphi$$

$$\triangle = \int_0^{\frac{\pi}{2}} d\theta \int_0^{2\pi} \sin\theta \, d\varphi = 2\pi$$

$$\bigcirc = 4\pi$$

step 1

Blackbody $\quad I_\lambda = \text{constant} \quad (\text{maximum})$

$$E_{b\lambda} = \frac{\int P_\lambda' \, d\Omega}{dA \, d\lambda} = \int_\triangle I_{b\lambda} \cos\theta \, d\Omega$$

$$= I_{b\lambda} \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\varphi$$

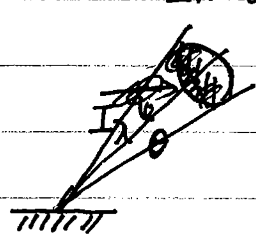
$$E_{b\lambda} = \pi I_{b\lambda}$$

$$\frac{P_\lambda}{dA \, d\lambda \, d\Omega} = \underbrace{I_{b\lambda} \cos\theta}$$

Lambert Law (surface)

4. Radiation pressure

Einstein: $\quad p = \frac{h}{\lambda}$



$\leadsto \frac{I_\lambda}{h\nu}$ ~~= flux~~ photon # per ~~unit time~~

$$\int_\triangle p \cdot \frac{I_\lambda}{h\nu} \cos\theta \, d\Omega$$

$$\frac{P_\lambda}{dA \, d\lambda} = I_\lambda \cos\theta \, d\Omega$$

$$\# = \frac{I_\lambda \cos\theta \, d\Omega}{h\nu}$$

Normal momentum $\int_\Omega \frac{h}{\lambda} \cdot \frac{I_\lambda \cos\theta\, d\Omega}{h\nu} \cdot \cos\theta$

$$= \frac{I_\lambda}{c} \int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$= \frac{2\pi I_\lambda}{c} \cdot \frac{1}{3} = \frac{2\pi I_\lambda}{3c}$$

$$= \frac{2}{3c} E_{b\lambda}.$$

Solar sail

5. Visible radiation   Human.

Certain fraction of intensity is observed as light

Luminous Intensity

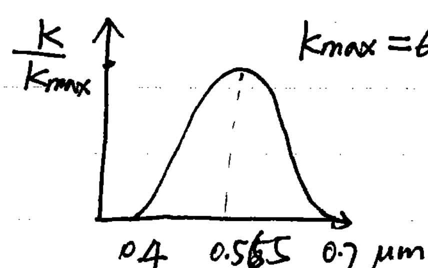
$$L_\lambda = k_\lambda I_\lambda$$

↑ luminous efficacy. $\frac{Lm}{W}$

$\frac{\text{Lumen}}{\Delta\lambda\, \Delta\Omega\, dA_\perp}$ [lm/m² sr μm]

└ normal to ray

$K_{max} = 683$ lm/W



Total

$$L = \int_0^\infty k_\lambda I_\lambda d\lambda \approx 286 \frac{lm}{W} \int_{0.4}^{0.7} I_\lambda d\lambda$$

60 W ⇒ 840 lm light bulb ⇒ $\int_{0.4}^{0.7} I_\lambda d\lambda =$

↳ 3 W ⇒ 5%

6. Properties of Surfaces (Ch. 1 & 3)

Emissivity

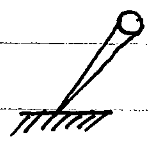
$$\epsilon = \frac{\text{energy emitted from a surface}}{\text{energy emitted by a black } \cancel{\text{body}} \text{ surface at same temp}}$$

ϵ'_λ — ~~directional~~ spectral directional

ϵ_λ — spectral hemispherical

ϵ' — total, directional

ϵ — total, hemispherical



$$\epsilon'_\lambda = \frac{I_\lambda(\theta,\varphi,T) \cos\theta \, dA \, d\lambda \, d\Omega}{I_{b\lambda}(\theta,\varphi,T) \cos\theta \, dA \, d\lambda \, d\Omega}$$

$$= \frac{I_\lambda(\theta,\varphi,T)}{I_{b\lambda}(\theta,\varphi,T)}$$

$$\epsilon_\lambda = \frac{\int I_\lambda \cos\theta \, d\Omega}{\int I_{b\lambda} \cos\theta \, d\Omega} = \frac{\int_\Delta \epsilon'_\lambda I_{b\lambda} \cos\theta \, d\Omega}{\pi I_{b\lambda}}$$

$$= \frac{1}{\pi} \int_\Delta \epsilon'_\lambda \cos\theta \, d\Omega$$

similarly $\quad \epsilon' = \frac{1}{n^2 \sigma T^4} \int_0^\infty \epsilon'_\lambda E_{b\lambda}(T,\lambda) \, d\lambda$

$$\epsilon(T) = \frac{1}{n^2 \sigma T^4} \int_0^\infty \epsilon_\lambda(T,\lambda) E_{b\lambda}(T,\lambda) \, d\lambda$$

gray emitter: $\quad \epsilon'_\lambda = \epsilon'$

diffuse emitter $\quad \epsilon'_\lambda = \epsilon_\lambda$

diffuse-gray emitter: $\quad \epsilon(T) = \epsilon_\lambda = \epsilon'_\lambda = \epsilon'$

Reflectivity

$$\rho = \frac{\text{reflected energy (power)}}{\text{incoming energy (power)}}$$

Absorptivity

$$\alpha = \frac{\text{absorbed energy}}{\text{incoming energy}}$$

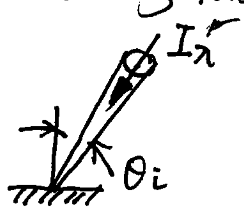
power absorbed per dA

$$\left\{ \begin{array}{l} \alpha_\lambda' = \dfrac{H_{a\lambda}'}{I_\lambda \cos\theta} = \dfrac{H_{a\lambda}'}{H_\lambda'} \\ \quad = \alpha_\lambda'(T, \theta_i, \varphi_i) \\ \alpha_\lambda \\ \alpha' \\ \alpha \end{array} \right.$$

Transmissivity

$$\tau = \frac{\text{transmitted energy}}{\text{incoming energy}}$$

Define Incoming Radiation power

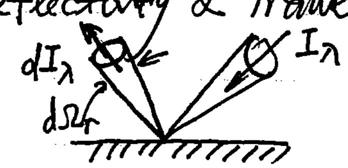


$$P_\lambda' = I_\lambda \cos\theta_i \, dA \, d\lambda \, d\Omega.$$

$$I_\lambda(T_s, \theta_i, \varphi_i, \lambda)$$

source temp. or

Reflectivity & Transmissivity



bi-directional reflectivity

incident reflected

$$\rho_\lambda''(\lambda, \hat{s}_i, \hat{s}_r) = \frac{dI_\lambda(\lambda, \hat{s}_i, \hat{s}_r)}{I_\lambda(\lambda, \hat{s}_i)\cos\theta_i \, d\Omega_i}$$

spectral directional-hemispherical reflectivity

$$\rho_\lambda'^{\cap} = \frac{\int_\cap dI_\lambda(\hat{s}_i, \hat{s}_r)\cos\theta_r \, d\Omega_r}{H_\lambda' \, d\Omega_i}$$

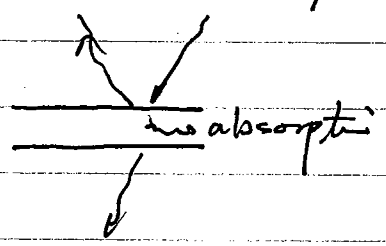
$$= \int_{2\pi} \rho''(\lambda, \hat{s}_i, \hat{s}_r) \cos\theta_r \, d\Omega_r$$

$$= \pi \rho_\lambda''$$ if diffuse reflector

other $\rho_\lambda^{\cap\prime}$ — spectral, hemispherical, directional reflectivity.

8 definitions: $\rho_\lambda'', \rho_\lambda'^{\cap}, \rho_\lambda^{\cap\prime}, \rho_\lambda \mid \rho'', \rho'^{\cap}, \rho^{\cap\prime}, \rho$

Transmissivity — Similar



energy balance

$$\rho + \alpha + \tau = 1$$

$$\rho_\lambda'^{\cap} + \alpha_\lambda' + \tau_\lambda'^{\cap} = 1.$$

opaque $\tau_\lambda'' = 0$

$$\rho + \alpha = 1.$$

we will learn how to calculate ρ & α for ideal surfaces.

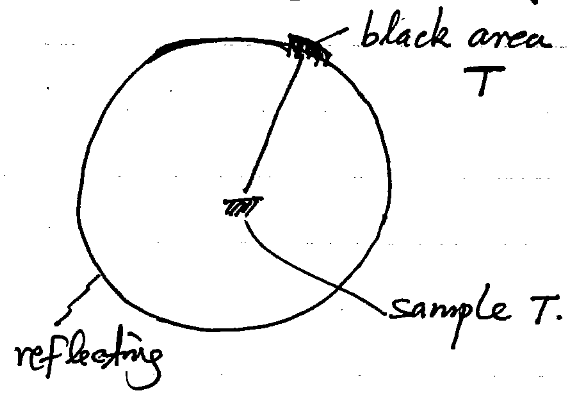
Diffuse absorber, gray absorber
Diffuse reflector

7. Kirchoff's Law

$$\epsilon_\lambda'(T) = \alpha_\lambda'(T)$$

sample temperature

Not ~~more~~ rigorous proof



only

only ϵ_λ'

α_λ' not zero

all other wave length & angle zero.

The only way to exchange energy is between ~~those~~ θ, φ, & λ.

Energy balance

$$\alpha_\lambda' I_{b\lambda} \cos\theta \, dA \, d\Omega \, d\lambda = \epsilon_\lambda' I_{b\lambda} \cos\theta \, dA \, d\Omega \, d\lambda$$

$$\Rightarrow \quad \alpha_\lambda' = \epsilon_\lambda'$$

Principle of detailed balance

2nd law of thermodynamics.

- Diffuse-gray surface, simple cases

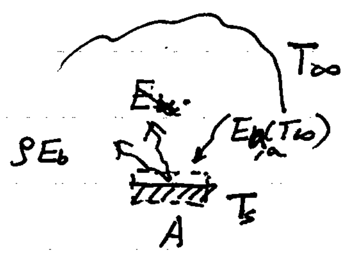
  diffuse-gray emitter : $\varepsilon'_\lambda = \varepsilon_\lambda = \varepsilon' = \varepsilon$

  diffuse-gray reflector : $\rho''_\lambda = \rho$

  Opaque

$$\rho + \alpha = 1$$

$$\alpha = 1 - \rho = \varepsilon$$

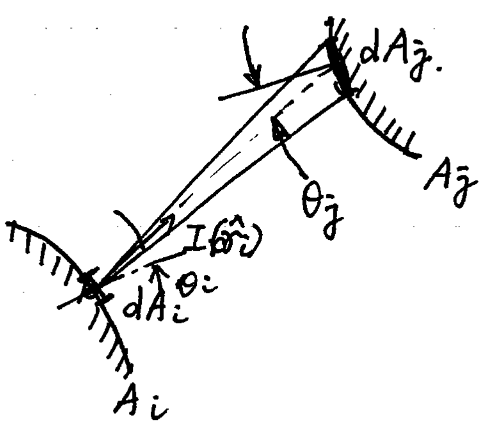


$$E = \varepsilon E_b$$

$$q = \varepsilon E_{b,s} - \alpha E_{b,a}$$

$$= \varepsilon \sigma (T_s^4 - T_\infty^4)$$

- Diffuse-gray surfaces, view factor.



$I(\vec{r}_i)$ isotropic

$\quad \llcorner T(\vec{r}_i)$

$$I = \frac{P'}{dA_{i\perp}\, d\Omega_{dA_i\, dA_j}}$$

P' — all the energy in the solid angle

$$P' = I\, dA_i \cos\theta_i \frac{dA_j \cos\theta}{R^2}$$

Total energy leaving dA_i $\quad \pi I dA_i$

Ratio

$$\frac{P'}{\pi I dA_i} = F_{dA_i - dA_j} = \frac{\cos\theta_i \cos\theta_j\, dA_j}{\pi R^2}.$$

$$L = \frac{\text{Power intercepted by } dA_j}{\text{total power leaving } dA_i}$$

"Intercepted ≠ absorbed"