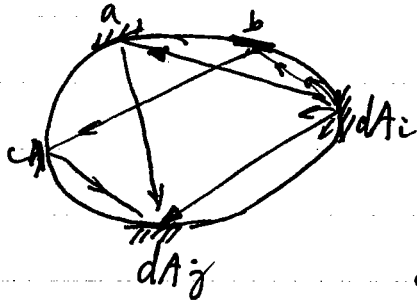


1. Review of last lecture.



- a. Radiation network method
 - b. Partially diffuse & specular } diffuse
- $$dF_{dA_i \rightarrow dA_j}^s = \frac{\text{Intercepted}}{\phi} \left\{ \begin{array}{l} \text{diffuse} \\ \text{specular} \end{array} \right.$$
- $\phi = P_{dA_i}^{(d)}$ (total diffuse)

$$dF_{dA_i \rightarrow dA_j}^s = dF_{dA_i \rightarrow dA_j} + \rho_a^s dF_{dA_i \rightarrow (a)} + \rho_b^s \rho_c^s dF_{dA_i \rightarrow (b,c)} + \dots$$

energy balance



$$q(\vec{r}) = \epsilon E_b(\vec{r}) - \alpha H(\vec{r})$$

$$= \epsilon(\nu) [E_b(\vec{r}) - H(\vec{r})]$$

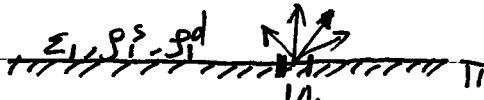
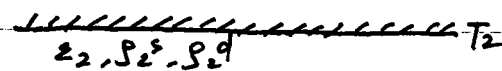
$$f(\vec{r}) = \cancel{\rho^d H} J^{tot} - H(\vec{r})$$

$$= \rho^d H + \rho^s H + \epsilon E_b - H$$

$$= \underset{\substack{\uparrow \\ \text{Diffuse part only}}}{J(\vec{r})} - [1 - \rho^s] H$$

$$H(\vec{r}) = \int_A J(\vec{r}') dF_{dA \rightarrow dA'}^s + H_o^s$$

Example:



2 infinite parallel plates

$$dF_{d1 \rightarrow 2}^{(s)}$$

$$= 1 + \rho_2^s \rho_1^s + (\rho_2^s \rho_1^s)^2 + \dots$$

$$= \frac{1}{1 - \rho_1^s \rho_2^s}$$

$$= F_{1 \rightarrow 2}^s$$

since every area behaves same.

reciprocity $F_{2 \rightarrow 1}^s$

$$F_{1 \rightarrow 2}^s > 1$$

but each time, absorbed $(1 - \rho_2^s)$

↳ specular part only.

$$(1 - \rho_2^s) F_{1 \rightarrow 2}^s \Rightarrow \text{fraction absorbed.}$$

↳ Summation rule $\sum_{j=1}^N (1 - \rho_j^s) F_{ij}^s = 1$

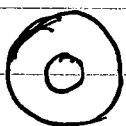
$$q_1 = J_1 - [1 - \rho_1^s] J_2 F_{1 \rightarrow 2}^s$$

$$q_2 = J_2 - [1 - \rho_2^s] J_1 F_{2 \rightarrow 1}^s$$

$$q_1 = \frac{\epsilon_1}{\rho_1^d} [(1 - \rho_1^s) E_{b1} - J_1]$$

$$q_2 = \frac{\epsilon_2}{\rho_2^d} [(1 - \rho_2^s) E_{b2} - J_2]$$

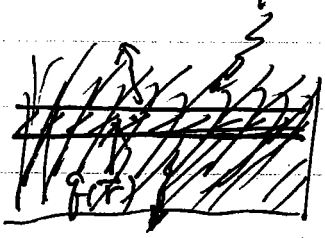
$$\Rightarrow q_1 = \frac{E_{b1} - E_{b2}}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad \text{— for both specular & diffuse}$$



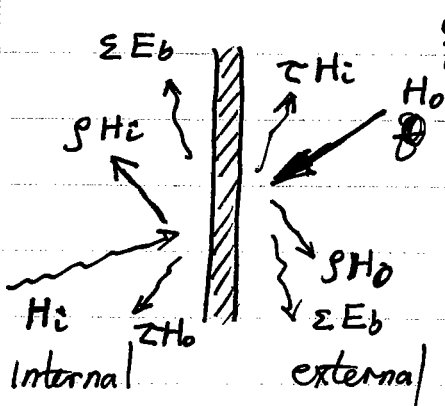
Concentric spheres/cylinders

$$q_1 = \frac{E_{b1} - E_{b2}}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \frac{A_1}{A_2} - \frac{A_1 A_2 \rho_2^s}{1 - \rho_2^s}}$$

* semi-transparent.



previous concepts are valid
only thing that needs to be
changed is energy balance



$$q_o = \Sigma E_b + \rho H_o + \tau H_i - H_o$$

leaving

$$q_i = \Sigma E_b + \rho H_i + \tau H_o - H_i$$

leaving

$$q = q_i + q_o$$

↑ supply

Additional sheet

* Non ~~gray~~ gray Surfaces

$$E_{b,\lambda} = \int_0^\infty E_{b,\lambda} d\lambda = \sum_{m=1}^M E_{b_j}^{(m)}$$

$$q = \int_0^\infty q_\lambda d\lambda = \sum_{m=1}^M q^{(m)}$$

M-bands, properties $\epsilon_\lambda, \alpha_\lambda, \dots$ constant in each ^{band}

$$\left. \begin{aligned} q_\lambda &= J_\lambda - H_\lambda \\ &= \epsilon_\lambda E_{b,\lambda} - \alpha_\lambda H_\lambda \end{aligned} \right\}$$

$$q_\lambda^r = \frac{E_{b,\lambda} - J_\lambda}{(1-\epsilon_\lambda)/\epsilon_\lambda}$$

$$q = \sum_m q^{(m)} = \sum_m (J_\lambda^{(m)} - H_\lambda^{(m)})$$

what is known

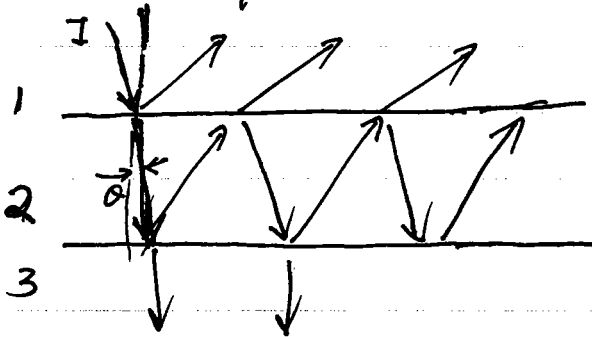
$$H_\lambda(\vec{r}) = \int J_\lambda(\vec{r}') dF_{dA-dA'}$$

$$H_\lambda^m = \int J_\lambda^m dF_{dA-dA'} + H_{o,\lambda}^m$$

Semitransparent sheet P. 96

$$I = I_0 e^{-\alpha x}$$

$$= I_0 e^{-\alpha d / \cos \theta}$$



$$R_{slab} = \rho_{12} + \rho_{23} (1 - \rho_{12}) \tau \cdot \rho_{23} \cdot \tau (1 - \rho_{21})$$

$$+ \rho_{23} \tau^2 (1 - \rho_{12})^2 \rho_{23} \tau (1 - \rho_{21})$$

$$+ \dots$$

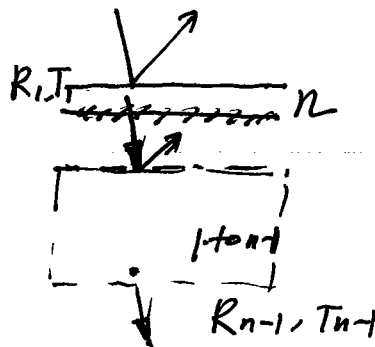
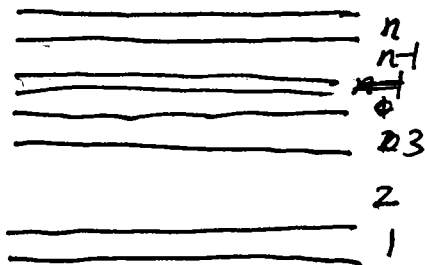
$$= \rho_{12} + \rho_{23} (1 - \rho_{12})^2 \tau^2 [1 + \rho_{12} \rho_{23} \tau^2 + (\rho_{12} \rho_{23} \tau^2)^2 + \dots]$$

$$= \rho_{12} + \frac{\rho_{23} (1 - \rho_{12})^2 \tau^2}{1 - \rho_{12} \rho_{23} \tau^2}$$

$$\tau_{slab} = \frac{(1 - \rho_{12})(1 - \rho_{23})\tau}{1 - \rho_{12} \rho_{23} \tau^2}$$

$$A_{slab} = 1 - R_{slab} - \tau_{slab}$$

Parallel sheet



Treat as one interface

$$R_n = R_{n-1} + \frac{T_{n-1}^2 R_n}{1 - R_{n-1} R_n}$$

Recursive formula

$$\Rightarrow q^{(m)} = J_{\text{ref}}^{(m)} - \left(\int J_{\lambda}^{(m)} dF_{dA-dA'} + H_0^{(m)} \right)$$

N surfaces

$$q_j^{(m)} = J_{\text{ref},j}^{(m)} - \sum_{i=1}^N J_{i,j}^{(m)} F_{i-j} + H_0^{(m)}$$

$m \times N$ equations

$$q_j^{(m)} = \frac{E_{b,j}^{(m)} - J_j^{(m)}}{(1 - \epsilon_j^{(m)}) / \epsilon_j^{(m)}} \quad m \times N \text{ equations}$$

$E_{b,j}(T_j) \Rightarrow$ need another N equations

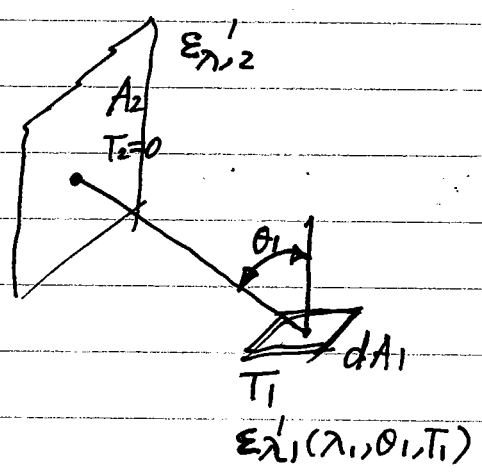
~~$q_j^{(m)} = \dots$~~

$$q_j = \sum q_j^{(m)} \quad N \text{ equations.}$$

* Monte Carlo Method (chapter 20)

Statistical method, random walk — every next step is independent of previous steps
 \hookrightarrow Markov Chain.

To get an idea, let's take the following example



How much radiative exchange $dQ_{1 \rightarrow 2}$?

(1) Total emitted power
 $dQ_{e,1} = \epsilon_1(T_1) \sigma T_1^4 dA_1$