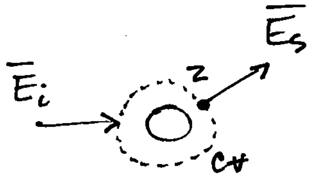
3/16/06 2.58

SCATTERING FROM PARTICLES



$$\bar{E}_2 = \bar{E}_i + \bar{E}_s$$

$$\bar{H}_2 = \bar{H}_i + \bar{H}_s$$

$$\langle \bar{S} \rangle = \frac{1}{2} Re\left(\bar{E} \times \bar{H}^*\right)$$

$$= \underbrace{\frac{1}{2} Re\left(\bar{E}_i \times \bar{H}_i\right)}_{\text{POWER IN}} + \underbrace{\frac{1}{2} Re\left(\bar{E}_s \times \bar{H}_s^*\right)}_{\text{POWER OUT}} + \frac{1}{2} Re\left(\bar{E}_i \times \bar{H}_s^* + \bar{E}_s \times \bar{H}_i^*\right)$$

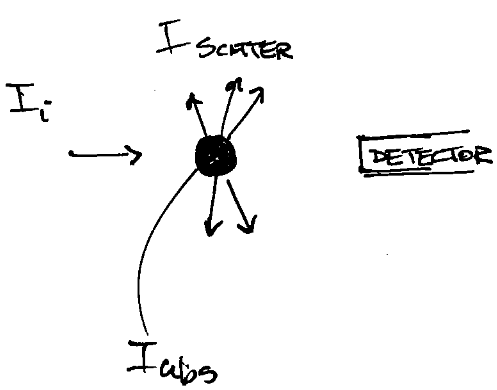
$$\int_A \langle \bar{S} \rangle \cdot d\bar{A} = \int_A \langle \bar{S}_i \rangle \cdot d\bar{A} + \int_A \langle \bar{S}_s \rangle \cdot d\bar{A} + \int_A \langle \bar{S}_e \rangle \cdot dA$$

$$-W_a = 0 + W_s - W_e \qquad W = [\text{POWER}]$$

$$W_e = W_a + W_s$$

W_e: POWER EXTINCTION

PHYSICAL PICTURE



DETECTOR WILL MEASURE LESS DUE TO INCOMING BEING SCATTERED AND ABSORBED.

3/16/06 2.58

CROSS-SECTIONS

SCATTERING CROSS-SECTION $C_s = \frac{W_s}{I_i}$

ABSORPTION " $C_a = \frac{W_a}{I_i}$

EXTINCTION " $C_e = \frac{W_e}{I_i}$

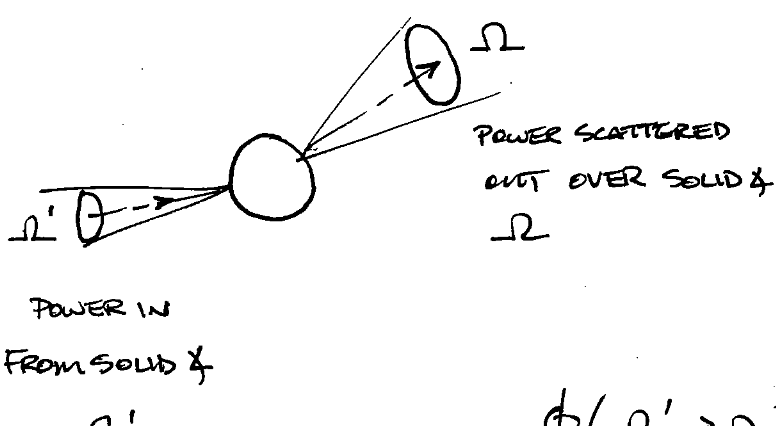
SCATTERING EFFICIENCY: $Q_s = \frac{C_s}{A_c}$ $(A_c = \pi r^2)$

ABSORPTION " : $Q_a = \frac{C_a}{A_c}$

EXTINCTION " : $Q_e = \frac{C_e}{A_c}$

ALBEDO: $\omega_0 = \frac{Q_s}{Q_e}$

PHASE FUNCTION (ϕ)



$$\phi(\Omega' \to \Omega) = \frac{\text{POWER SCATTERED INTO } \Omega}{\text{ISOTROPIC CASE}}$$

3/16/06 2.58

ISOTROPIC SCATTERING

$$\phi(\Omega' - \Omega) = 1$$

$$\frac{1}{4\pi}\int \phi(\Omega' \to \Omega)\, d\Omega = 1$$

ALL RESULTS THUSFAR ARE FOR ANY GEOMETRY, NOT JUST SPHERES

---

* SPHERICAL PARTICLES *

MIE THEORY (1908, GUSTOV MIE)

$$Q_e = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}\{a_n + b_n\}$$

$$Q_s = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2 + |b_n|^2\right)$$

$$a_n = f_1[\psi_n, \xi_n, m, x]$$

$$b_n = f_2[\qquad\qquad]$$

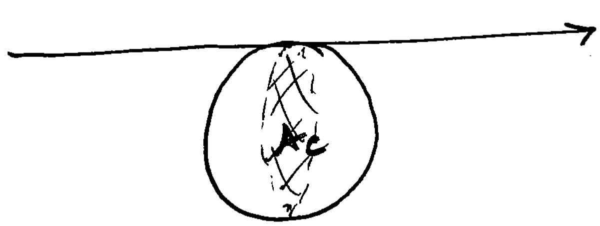
3/16/06

ψ_n, ξ_n } RICATTI-BESSEL FUNCTIONS

$$\psi_{n+1}(x) = \frac{2n+1}{x}$$

$x \equiv$ SIZE PARAMETER, $x = \frac{2\pi r}{\lambda_0}$ λ_0 IS WAVELENGTH IN SURROUNDING MEDIA

$$m = \frac{N_1}{N_0} \quad \frac{\text{(PARTICLE REFRACTIVE INDEX)}}{\text{(REFRACTIVE INDEX OF MEDIUM)}}$$



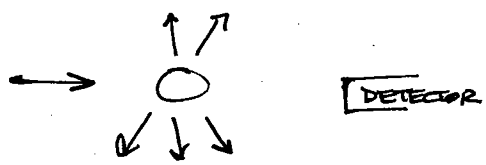
$\frac{\lambda}{d}$

- BIG PARTICLE SCATTERING SHOULD BE $2A_c$, BUT NOT REALLY TRUE

3/16/06 2.58

VERY SMALL PARTICLE

$Q_a > 1$ $\qquad \varepsilon_\lambda = \dfrac{P_{e\lambda}}{4\pi r^2 E_{b\lambda}}$ CAN BE > 1 → VIOLATES PLANCK'S LAW, BUT PLANCK SAYS HIS LAW IS NOT VALID IN THIS CASE.



★ DETECTOR SHOULD BE "SMALL", SO AS NOT TO PICK-UP SIDE SCATTERED POWER

POWER MEASURED @ DETECTOR

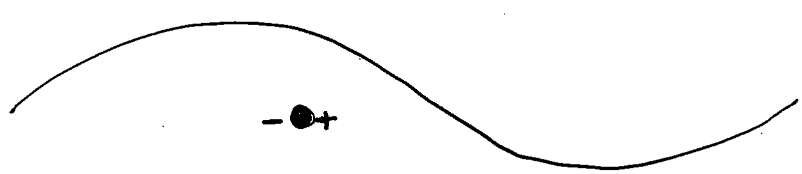
$$U = I_i (A_c - C_e)$$

RAYLEIGH SCATTERING $(X \ll 1)$

$$Q_s = \frac{8}{3}\left|\frac{m^2-1}{m^2+2}\right|^2 x^4 \sim \frac{r^4}{\lambda^4} \qquad C_s \sim r^6 \sim V^2$$

$$Q_e = 4\, I_m\left(\frac{m^2-1}{m^2+2}\right) x \sim \frac{r}{\lambda} \qquad C_a \sim r^3 \sim V$$

$$\approx Q_a$$

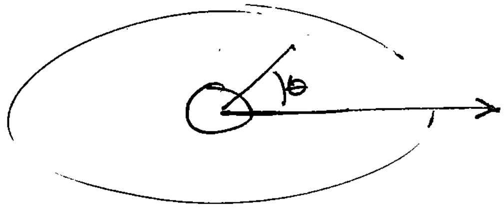


INDUCED POLARIZATION

$$\vec{P} = \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right) r^3 \vec{E}_{INDUCE}$$

$\vec{D} = k$ ...

RAYLEIGH PHASE FUNCTION



$$\phi(\theta) = \frac{3}{4}\left(1 + \cos^2\theta\right)$$

RAYLEIGH – GANS LIMIT $|m-1| \ll 1$

GEOMETRIC LIMIT $x|m-1| \ll 1$

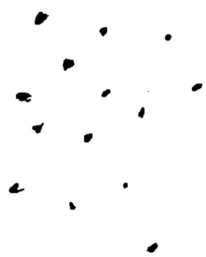
RAINBOW PHENOMENON ⟹ RELATED TO PHASE FUNCTION

H_2O PARTICLE

VARIOUS ANGLES SCATTER DIFFERENT WAVELENGTHS OF LIGHT FROM THE H_2O PARTICLE MORE STRONGLY OR WEAKLY

$$\cos\theta_l = \sqrt{\frac{n^2-1}{l(l+2)}}, \quad n = 1.33, \quad \theta_l = 42^\circ$$

SYSTEM OF PARTICLES



$$N_T \triangleq \frac{\text{PARTICLE \# DENSITY}}{\text{VOL.}}$$

UNIFORM SIZE SCATTERING COEFF.

$$\sigma_{s\lambda} = C_s N_T \quad - \quad \frac{1}{m}$$

ABS. COEF. $\quad \kappa_\lambda = C_a N_T$

EXT. COEF $\quad \beta_\lambda = C_e N_T = \sigma_{s\lambda} + \kappa_\lambda$