Cavitation Notes

ref: PNA pages 181-183 handout

$p_0 = \text{uniform\_stream\_total\_pressure}$ $p_1 = \text{pressure\_at\_arbitrary\_point}$

$V_0 = \text{uniform\_stream\_velocity}$ $V_1 = \text{velocity\_at\_arbitrary\_point}$

$$q = \frac{1}{2}\cdot\rho\cdot V_0^{\ 2} = \text{dynamic\_or\_stagnation\_or\_ram\_pressure}$$

$$p_0 + \frac{1}{2}\cdot\rho\cdot V_0^{\ 2} = \text{constant}$$ Bernoulli

$$p_0 := \text{constant} - \frac{1}{2}\cdot\rho\cdot V_0^{\ 2} \qquad p_1 := \text{constant} - \frac{1}{2}\cdot\rho\cdot V_1^{\ 2}$$

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for propeller immersion, measured at radius r, minimum p_0 is obtained from ...

$$p_0 = p_a + \rho\cdot g\cdot h - \rho\cdot g\cdot r$$

$p_a = \text{atmosphere}$

$h = \text{shaft\_centerline\_immersion}$

$\rho\cdot g\cdot r$ accounts for minimum when r vertical up

V_0 estimated as (VA^2+(ω*r)^2)$^{0.5}$

if p_1 => p_v = vapor pressure, cavitation occurs

define:

$$\sigma_L = \text{local\_cavitation\_number} = \frac{p_a + \rho\cdot g\cdot h - \rho\cdot g\cdot r - p_v}{\frac{\rho}{2}\cdot\left(V_A^{\ 2} + \omega^2\cdot r^2\right)}$$

and if pressure REDUCTION / q >= σ_L cavitation occurs

early criteria (Barnaby) suggested limiting average thrust per unit area to certain values (76.7 kN/m^2 = 10.8 psi) for tip immersion of 11 in increasing by 0.35 psi (unit conversions don't match up)

$$76.7\frac{kN}{m^2} = 11.124\,psi$$

earlier PNA (1967) stated Barnaby suggested 11.25 psi

can calculate pressure distributions around blade so can calculate local cavitation situation

early in propeller design, want blade area to avoid cavitation (more blade area, less pressure per unit area for given thrust)

Burrill ((1943) "Developments in Propeller Design and Manufacture for Merchant Ships", Trans. Institute of Marine Engineers, London, Vol. 55) proposed guidance as follows:
limit thrust (coefficient) to a certain value depending on cavitation number at the 0.7 radius

$\tau_c = \text{coefficient\_expressing\_mean\_loading\_on\_blades}$

$$\tau_c = \frac{\frac{T}{A_P}}{\frac{1}{2}\cdot\rho\cdot V_R^{\ 2}}$$

$T = \text{thrust}$ $\rho = \text{water\_density}$

$A_P = \text{projected\_area}$ $V_R = \text{relative\_velocity\_of\_water\_at\_0\_7\_radius}$

can estimate projected area from

$$\frac{A_P}{A_D} = 1.067 - 0.229\cdot\frac{P}{D}$$

from Taylor S & P page 91 P/D from 0.6 to 2.0 elliptical bladed prop, hub = 0.2 D and as usual ... $T = \frac{P_E}{(1-t)\cdot V} = \frac{P_D\cdot\eta_D}{(1-t)\cdot V}$

P_D = delivered_power $\quad R_T = T\cdot(1-t)$

P_E = effective_power $\quad P_E = R_T\cdot V$

η_D = quasi_propulsive_coefficient $= \frac{P_E}{P_D} = \eta_H\cdot\eta_R\cdot\eta_o \qquad \eta_H = \frac{1-t}{1-w}$

this parameter is plotted versus $\sigma_{0.7\cdot}$ cavitation number at 0.7*r using relative velocity at 0.7*r and pressure at CENTERLINE

$\rho := 1.0259\cdot 10^3 \frac{kg}{m^3} \qquad \rho = 1.99057 \frac{slug}{ft^3}$

$V_A = \frac{m}{sec} \qquad h = m$

$\sigma_{0.7\cdot} = \frac{p_0 + \rho\cdot g\cdot h - p_v}{\frac{1}{2}\cdot\rho\cdot\left[V_A^2 + (0.7\cdot\pi\cdot n\cdot D)^2\right]} = \frac{188.2 + 19.62\cdot h}{V_A^2 + 4.836\cdot(n\cdot D)^2}$

$D = m \qquad n = sec^{-1}$

units in PNA (61) approximation SI pv apparently ~ 0.69 psi (90 degF)

$\sigma_{0.7\cdot} = \frac{2026 + 64.4\cdot}{V_A^2 + 4.836\cdot(n\cdot D)^2}$ in US units

Carmichael correlation

$C := \frac{\tau_c + 0.3064 - 0.523\cdot\sigma^{0.2}}{0.0305\cdot\sigma^{0.2} - 0.0174}$

C = cavitation % $\quad \tau_c$ as above

σ at 0.7 radius as above (centerline immersion)

example numbers

for C <= 25 (%) or so

$\sigma := 0.4 \quad \tau_c := 0.2$

$C := \frac{\tau_c + 0.3064 - 0.523\cdot\sigma^{0.2}}{0.0305\cdot\sigma^{0.2} - 0.0174}$

$C = 8.88$ % cavitation

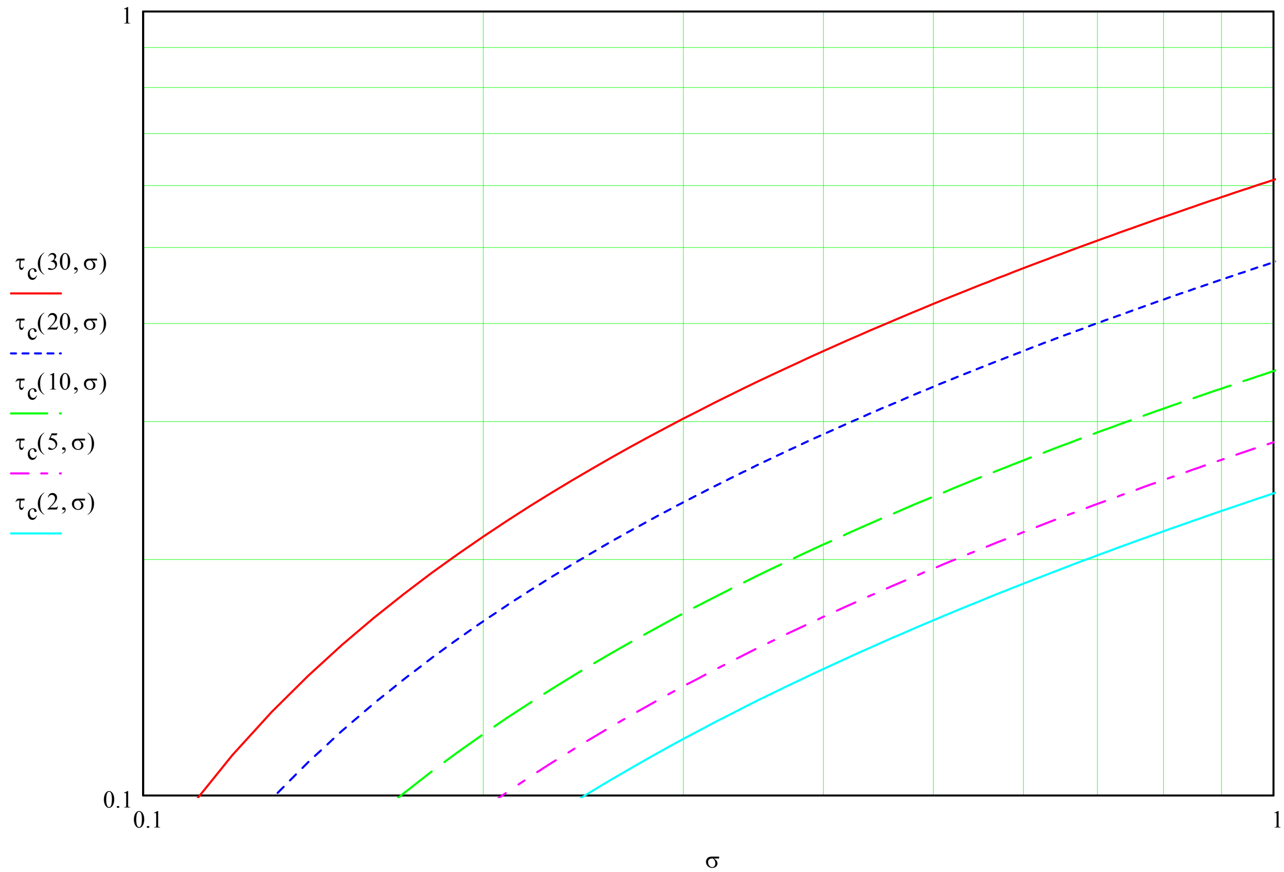
$\sigma := 0.1, 0.11 .. 2$ or ... $\tau_c(C,\sigma) := C\cdot\left(0.0305\cdot\sigma^{0.2} - 0.0174\right) + 0.523\cdot\sigma^{0.2} - 0.3064$ see plot below

this can be carried further into ...

$$\tau_c = \frac{\frac{T}{A_P}}{\frac{1}{2}\cdot\rho\cdot V_R^2} = \left[C\cdot\left(0.0305\cdot\sigma^{0.2} - 0.0174\right) + 0.523\cdot\sigma^{0.2} - 0.3064\right]$$

$$A_P = \frac{T}{\frac{1}{2}\cdot\rho\cdot V_R^2\cdot\left[C\cdot\left(0.0305\cdot\sigma^{0.2} - 0.0174\right) + 0.523\cdot\sigma^{0.2} - 0.3064\right]}$$

A_p = minimum_area_for_specified_cavitation



Carmichael correlation valid only for C <= 25 %. 30 % shown to indicate over estimates compared with fig 45 page 182 of PNA

for example from prop_design_notes

derived above

$rpm := min^{-1}$ $kt := 1.688\,\frac{ft}{s}$

$T := 278000lbf$ $V_A := 14kt$

$p_0 = 14.696\,psi$

$n := 218rpm$

$p_v = 0.694\,psi$

$D := 15ft$

$P\_over\_D := 0.8$

$h := 10ft$

$$\sigma_{0.7\cdot r} = \frac{p_0 + \rho\cdot g\cdot h - p_v}{\frac{1}{2}\cdot\rho\cdot\left[V_A^{\,2} + (0.7\cdot\pi\cdot n\cdot D)^2\right]} = \frac{188.2 + 19.62\cdot h}{V_A^{\,2} + 4.836\cdot(n\cdot D)^2}$$

$V_A = \frac{m}{sec}$ $D = m$ $h = m$ $n = sec^{-1}$

$V_A = 7.203\,\frac{m}{s}$ $D = 4.572\,m$ $h = 3.048\,m$ $n = 3.633\,\frac{1}{s}$

$$\sigma := \frac{p_0 + \rho\cdot g\cdot h - p_v}{\frac{1}{2}\cdot\rho\cdot\left[V_A^{\,2} + (0.7\cdot\pi\cdot n\cdot D)^2\right]}$$

$\sigma = 0.179$

$$\frac{188.2 + 19.62\cdot 3.048}{7.203^2 + 4.836\cdot(3.633\cdot 4.572)^2} = 0.179$$

using SI approximation

consider % cavitation in steps of 5%

$C := 5, 10 .. 25$ $V_R := \left[V_A^{\,2} + (0.7\pi n\cdot D)^2\right]^{0.5}$ $V_R = 37.234\,\frac{m}{s}$

$$A_P(C) := \frac{T}{\frac{1}{2}\cdot\rho\cdot V_R^{\,2}\cdot\left[C\cdot\left(0.0305\cdot\sigma^{0.2} - 0.0174\right) + 0.523\cdot\sigma^{0.2} - 0.3064\right]}$$

$$\frac{A_P}{A_D} = 1.067 - 0.229\cdot\frac{P}{D}$$ assume AD ~ AE

$$A_E(C) := \frac{A_P(C)}{1.067 - 0.229\cdot P\_over\_D}$$

cavitation % — estimated minimum EAR to avoid

$C =$	$A_P(C) =$ (m^2)	$A_E(C) =$ (m^2)	$\frac{A_E(C)}{\pi\cdot\frac{D^2}{4}} =$
5	20.367	23.045	1.404
10	16.333	18.48	1.126
15	13.632	15.425	0.94
20	11.698	13.236	0.806
25	10.245	11.592	0.706

supercavitating τ_c σ to the left. σ very low
cavitation % is 100

$h = 3.048\ m$ $D = 4.572\ m$ $V_A = 7.203\ \frac{m}{s}$ $n = 218\ \frac{1}{min}$ $0.7\cdot\pi\cdot n\cdot D = 36.531\ \frac{m}{s}$

$V_A := 10\ \frac{m}{s}$ $n := 1000rpm$ $0.7\cdot\pi\cdot n\cdot D = 167.573\ \frac{m}{s}$

$$\sigma_{0.7\cdot r} = \frac{p_0 + \rho\cdot g\cdot h - p_v}{\frac{1}{2}\cdot\rho\cdot\left[V_A^{\,2} + (0.7\cdot\pi\cdot n\cdot D)^2\right]}$$

$$\sigma := \frac{p_0 + \rho\cdot g\cdot h - p_v}{\frac{1}{2}\cdot\rho\cdot\left[V_A^{\,2} + (0.7\cdot\pi\cdot n\cdot D)^2\right]}$$

$\sigma = 8.8\times 10^{-3}$

to avoid 25% cavitation $C := 25$

$$\tau_c(C,\sigma) := C\cdot\left(0.0305\cdot\sigma^{0.2} - 0.0174\right) + 0.523\cdot\sigma^{0.2} - 0.3064$$

$\tau_c(C,\sigma) = -0.243$

off the scale hence supercavitating propellers correlation not valid but trend is ok