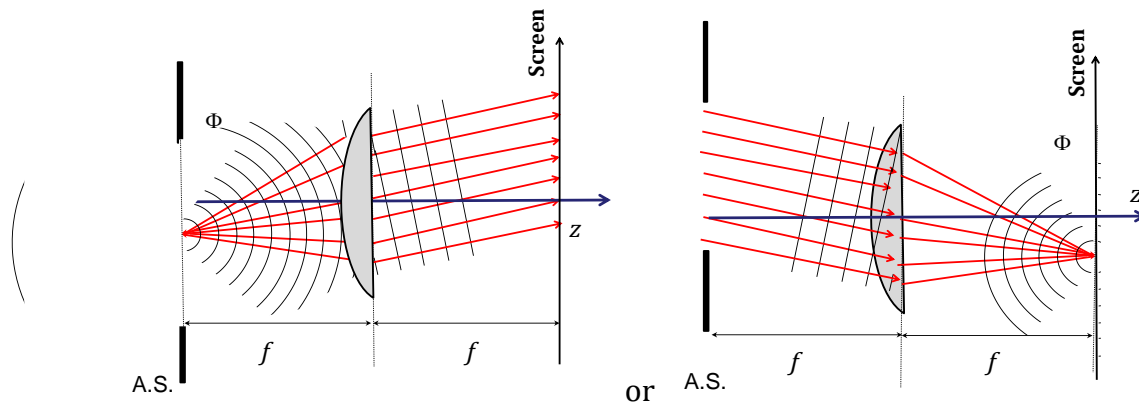


**Outline:**

- A. Imaging with coherent light
- B. Optical Spatial Filtering
- C. The significance of PSF and ATF, and effect of coherence
- D. Phase Contrast Imaging: Zernike and Schlieren methods

A. Imaging with Coherent Light

Recap: a convex lens conduct Fourier Transform at the two focal planes:



The two pictures above are interpretations of the same physical phenomenon. On the left, the transparency is interpreted as a superposition of “spherical wavelets.”

Each spherical wavelet is collimated by the lens and contributes to a plane wave at the output, propagating at the appropriate angle (scaled by  $f$ )

On the right, the transparency is interpreted in the Fourier sense as a superposition of plane waves (“spatial frequencies.”) Each plane wave is transformed to a converging spherical wave by the lens and contributes to the output, at distance  $f$  to the right of the lens, a point image that carries all the energy that departed from the input at the corresponding spatial frequency.

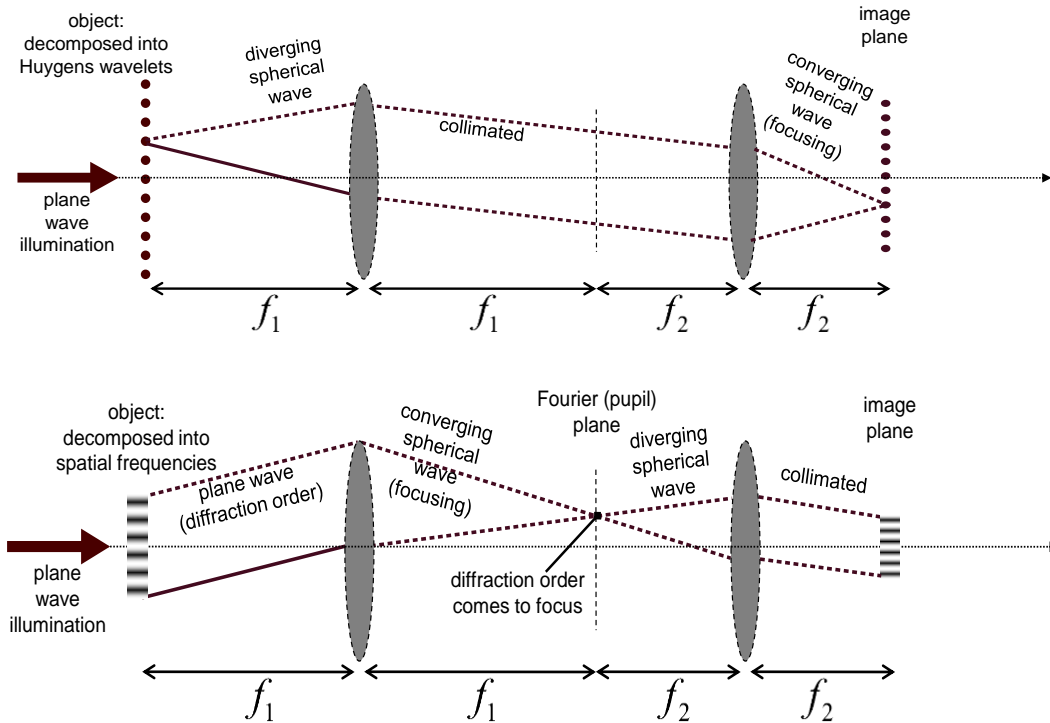
From the front focal plane to the back focal plane:

$$E_{out}(x', y') \approx \iint E_{in}(x, y) \exp\left\{\frac{-ik[x'x + y'y]}{f}\right\} dx dy \quad (1)$$

We see that:  $k_x = x' \frac{k}{f}$ ,  $k_y = y' \frac{k}{f}$  or

$$x' = k_x \frac{f}{k}, y' = k_y \frac{f}{k} \quad (2)$$

By cascading two lenses together, we can reveal **Abbe's theory** of imaging process:



Ideally, applying two forward Fourier transforms recovers the original function of the object field, with a reversal in the coordinates:

$$E_{image}(x'', y'') \approx \iint E(x', y') \exp\left\{\frac{-ik[x'x'' + y'y'']}{f_2}\right\} dx' dy' \quad (3)$$

Using  $x' = k_x \frac{f_1}{k}, y' = k_y \frac{f_1}{k}$

$$E_{image}(x'', y'') \approx \left(\frac{f_1}{k}\right)^2 \iint E(x', y') \exp\left\{-i \frac{f_1}{f_2} [k_x x'' + k_y y'']\right\} dk_x dk_y \quad (4)$$

$$\text{Let } -\frac{f_1}{f_2} x'' = x, -\frac{f_1}{f_2} y'' = y, \quad (5)$$

$$E_{image}(x'', y'') \propto \mathcal{F}\left(\mathcal{F}\left(E_{object}(x, y)\right)\right) = E_{object}\left(-\frac{f_2}{f_1} x, -\frac{f_2}{f_1} y\right) \quad (6)$$

Potentially, the magnification ratio  $M = f_2/f_1$  can be arbitrarily large. This however does not mean that the microscope is able to resolve arbitrarily small objects. The finite size of the aperture stop, and the corresponding transmission  $AS(x', y')$  will contribute to the above Fourier transforms:

$$AS(x', y') = AS\left(k_x \frac{f_1}{k}, k_y \frac{f_1}{k}\right) \quad (7)$$

$$E_{image}(x'', y'') \approx \mathcal{F} \left( AS(k_x \frac{f_1}{k}, k_y \frac{f_1}{k}) \times \mathcal{F} (E_{object}(x, y)) \right)$$

$$E_{image}(x'', y'') \approx E_{object} \left( -\frac{f_2}{f_1} x, -\frac{f_2}{f_1} y \right) \otimes \mathcal{F} \left[ AS \left( k_x \frac{f_1}{k}, k_y \frac{f_1}{k} \right) \right] \quad (8)$$

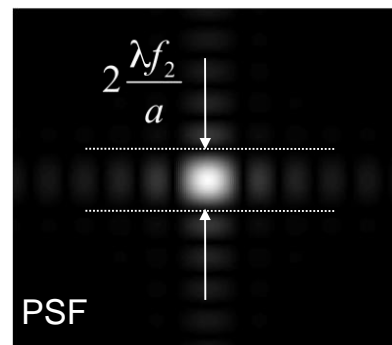
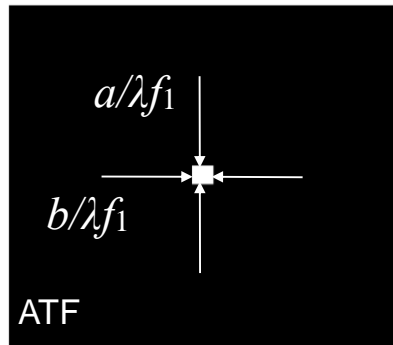
**Note:** In Goodman's book, the term  $AS(k_x \frac{f_1}{k}, k_y \frac{f_1}{k})$  is called Amplitude Transfer Function(ATF), and its Fourier transform,  $\mathcal{F} [AS(k_x \frac{f_1}{k}, k_y \frac{f_1}{k})]$  is called Point Spread Function(PSF) (since it is the spread of an ideal point source  $\delta(x, y)$  at the image).

Worked Examples:

1) Rectangle apertures:

$$ATF = \text{rect} \left( \frac{f_1 k_x}{ak} \right) \text{rect} \left( \frac{f_1 k_y}{bk} \right) \quad (9)$$

$$\begin{aligned} PSF(x'', y'') &\approx \left( \frac{f_1}{k} \right)^2 \iint \text{rect} \left( \frac{f_1}{ak} k_x \right) \text{rect} \left( \frac{f_1}{bk} k_y \right) \exp \left\{ -i \frac{f_1}{f_2} [k_x x'' + k_y y''] \right\} dk_x dk_y \\ PSF(x'', y'') &\approx \left[ \text{sinc} \left( \frac{ak}{f_1} x'' \right) \right] \left[ \text{sinc} \left( \frac{bk}{f_1} y'' \right) \right] \\ &= \left[ \text{sinc} \left( -\frac{ak}{f_2} x'' \right) \right] \left[ \text{sinc} \left( -\frac{bk}{f_2} y'' \right) \right] \end{aligned} \quad (10)$$

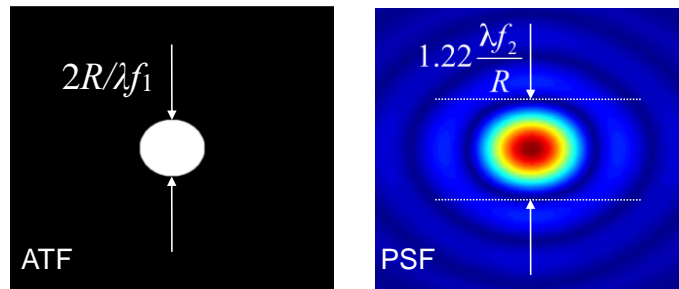


2) Circular apertures:

$$ATF = circ\left(\frac{f_1}{Rk}\sqrt{k_x^2 + k_y^2}\right) \quad (11)$$

$$PSF(x'', y'') \approx \left(\frac{f_1}{k}\right)^2 \iint circ\left(\frac{f_1}{Rk}\sqrt{k_x^2 + k_y^2}\right) \exp\left\{-i\frac{f_1}{f_2}[k_x x'' + k_y y'']\right\} dk_x dk_y$$

$$PSF(x'', y'') \approx a^2 J_{inc} \left(\frac{Rk}{f_1} \sqrt{x''^2 + y''^2}\right) = a^2 J_{inc} \left(\frac{Rk}{f_2} \sqrt{x''^2 + y''^2}\right) \quad (12)$$



### B. Optical Spatial Filtering

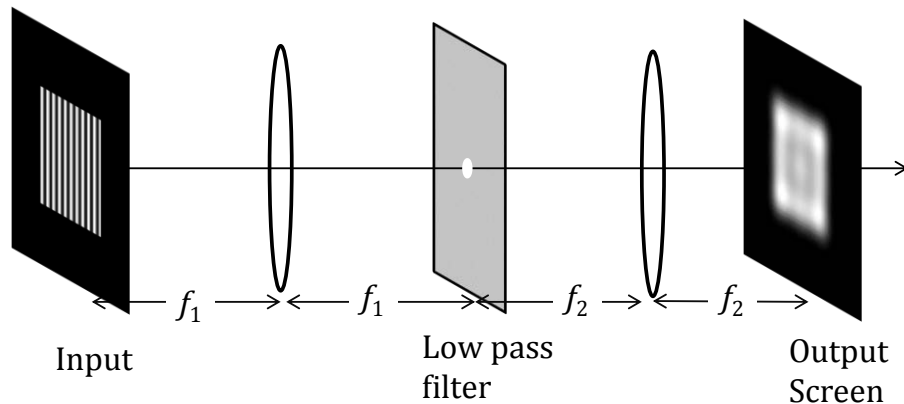
Spatial Filtering is a technique to process signals in an optical way, where the irradiance content in the Fraunhofer plane is manipulated to control the irradiance pattern in the image plane. A digital element to provide Spatial Filtering in a dynamical way is coined as a *spatial light modulator* (SLM). SLM consists of an array of pixels, each capable of controlling the amplitude or phase of the illuminating field. For example, liquid crystal SLMs control the amplitude and phase of the transmitted or reflected light. Likewise, TI's DLP SLM uses arrays of deformable micromirrors made by MEMS technology to adjust the amplitude and phase of the reflected light.

Examples of spatial light modulators removed due to copyright restrictions.

The basic setup for optical spatial filtering is a telescopic lens system, consisting of two lenses. Quite often, we can assume that the two lenses have the same focal

length  $f$  for simplicity. Then the distance from the object to the processed image is  $4f$ . For this reason such a system is called the 4-F setup for spatially filtering an image.

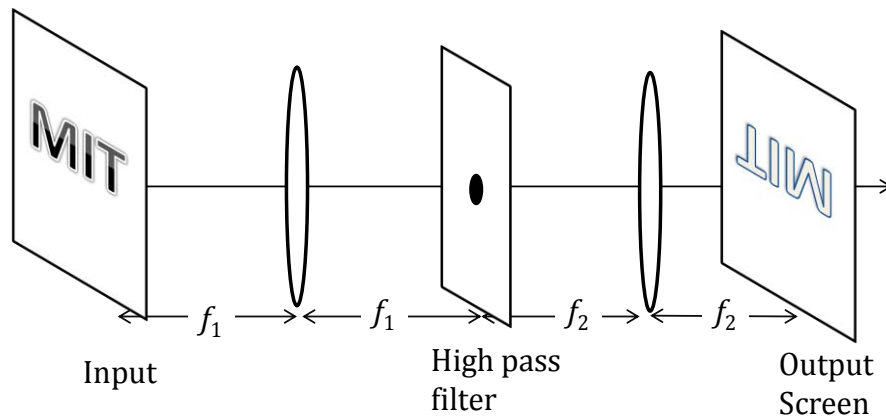
The following examples are typical image processing setup using various types of spatial frequency filters:



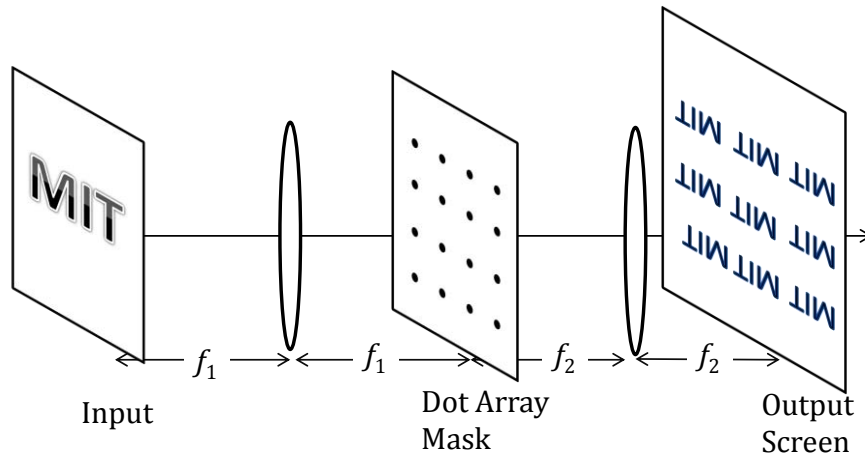
(a) **Low pass filter:** A circular aperture in the Fourier plane will block the high spatial frequencies and pass the low frequency ones. If the filter is an aperture of diameter  $a$ , the cutoff frequency is given by the condition:

$$k_{cutoff} = \frac{ka}{2f_1} \quad (13)$$

That is, features smaller than the length scale of  $2\lambda \frac{f_1}{a}$  is removed from the image.



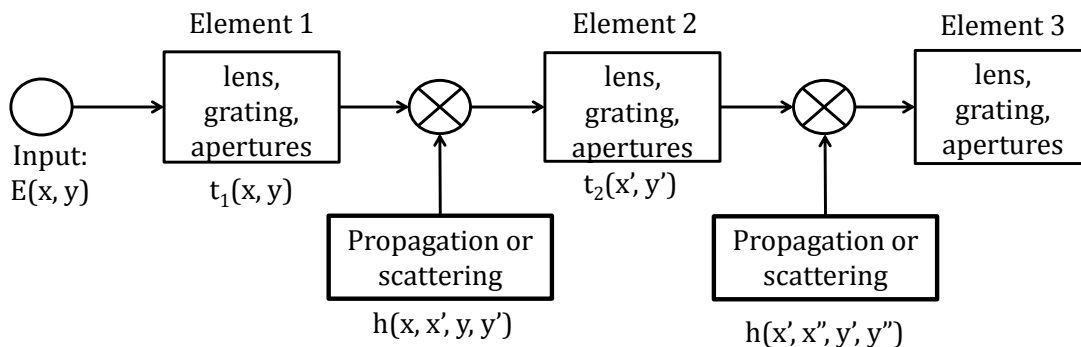
(b) **High pass filter:** A circular absorbing disk in the Fourier plane does the opposite to low pass filter. Features larger than the length scale of  $2\lambda \frac{f_1}{a}$  are removed from the image. This filter is also called a dark field filter in the microscopy and used frequently in materials science, as it allows only light scattered from sharp edges (such as grain boundaries) to pass through the filter.



(c) Step and Repeat operation

C. The significance of PSF and ATF

The theory of optical imaging and **communication** has a lot in common. The above imaging process might be modeled with an equivalent circuit. (Such analogy has stimulated research and development for basic operation such as multiplication, division, differentiation, correlation, etc. The application of the optical processing can be found in a variety of fields, such as, pattern recognition, computer aided vision, computed tomography, and image improvement. )

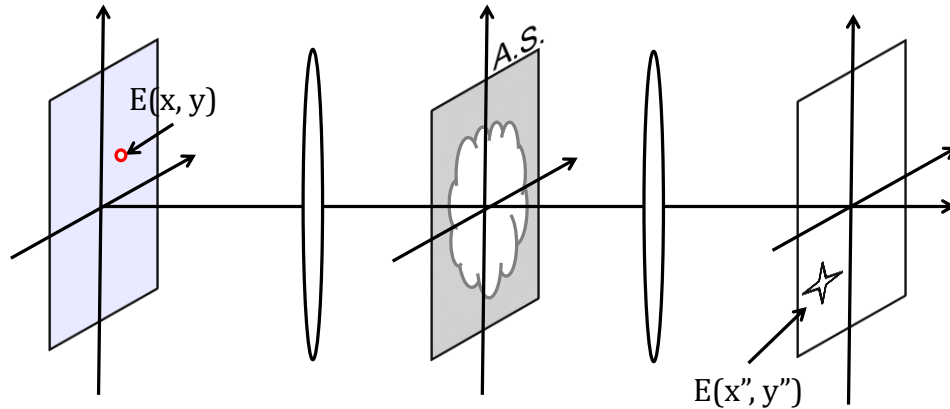


The typical distance between lenses and apertures are not large to meet Fraunhofer condition. Instead, we can use the following **Fresnel propagator**:

$$h(x, y, x', y', z) = \frac{\exp(ikz)}{z} \exp(ik \frac{(x'-x)^2 + (y'-y)^2}{2z}) \tag{14}$$

In coherent illumination, the point spread function (PSF) describes the response of the lens system to an impulse  $\delta(x, y)$  at the input field. If the

system is *shift-invariant*, then the observed image is a sum of all the field  $E(x'', y'')$  contributed by the spread of each point  $(x, y)$  from the object.



- Under (spatially) **coherent** illumination, the image field is a convolution of object field with point spread function (PSF). Correspondingly, the Amplitude Transfer Function (ATF) is the Fourier transform of PSF:

$$E_{image}(x'', y'') = E_{object} \left( -\frac{f_2}{f_1} x, -\frac{f_2}{f_1} y \right) \otimes PSF(x, y) \quad (15)$$

$$ATF(k_x, k_y) = \iint PSF(x, y) \exp(ik_x x + ik_y y) dx dy \quad (16)$$

e.g. ATF of single square aperture:

$$ATF = \text{rect} \left( \frac{f_1 k_x}{ak} \right) \text{rect} \left( \frac{f_1 k_y}{bk} \right)$$

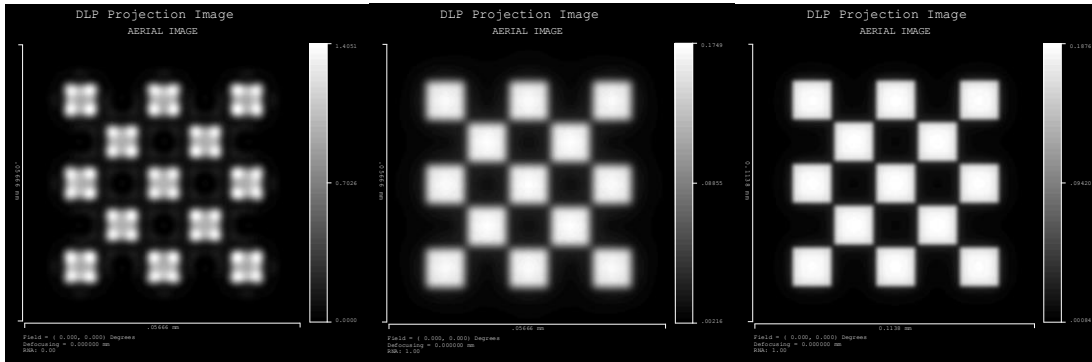
- Under (spatially) **incoherent** illumination, the image intensity is a convolution of object intensity with intensity of point spread function (iPSF= $|PSF|^2$ ). Correspondingly, the (complex) Optical Transfer Function (OTF) is the Fourier transform of iPSF:

$$I_{image}(x'', y'') = I_{object} \left( -\frac{f_2}{f_1} x, -\frac{f_2}{f_1} y \right) \otimes |PSF(x, y)|^2 \quad (17)$$

$$OTF(k_x, k_y) = \iint |PSF(x, y)|^2 \exp(ik_x x + ik_y y) dx dy = ATF \otimes ATF \quad (18)$$

e.g. OTF of single square aperture:

$$|OTF| = \Lambda \left( \frac{f_1 k_x}{ak} \right) \Lambda \left( \frac{f_1 k_y}{bk} \right) \quad (19)$$



Simulated intensity pattern of a 5x5 checkerboard illuminated by a light source with different coherence. (left)100% coherent; (middle)50% coherent; (right) non-coherent.

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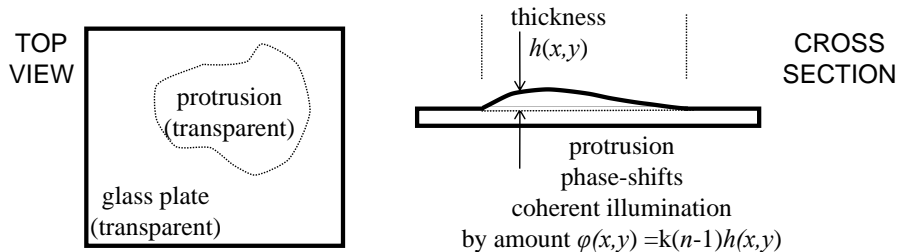
**D. More specific examples on Coherent Imaging**

**a. Zernike Phase-Contrast Imaging**

Zernike’s Phase Contrast is commonly used in biological microscopy to view transparent objects such as cellular membranes (that would otherwise require staining).

Let’s consider a transparent object with a small phase shift in the following form:

$$t(x, y) = \exp(ik(n - 1)h(x, y)) \approx 1 + ik(n - 1)h(x, y) \tag{20}$$



This model is often useful for imaging biological objects (cells, etc.)

When the transparent object is uniformly illuminated by a plane wave, the transmitted intensity is close to unity, leaving very low contrast. The idea behind the Zernike method starts with the observation that the unity part is the **dc** component in the Fourier plane, while  $\phi(x, y) = k(n - 1)h(x, y)$  represents a spatial distribution in the Fourier spectrum.

So what if we modify one of these to prevent the cancellation? Specifically, let’s try a  $\pi/2$  phase shift of the **dc** component:

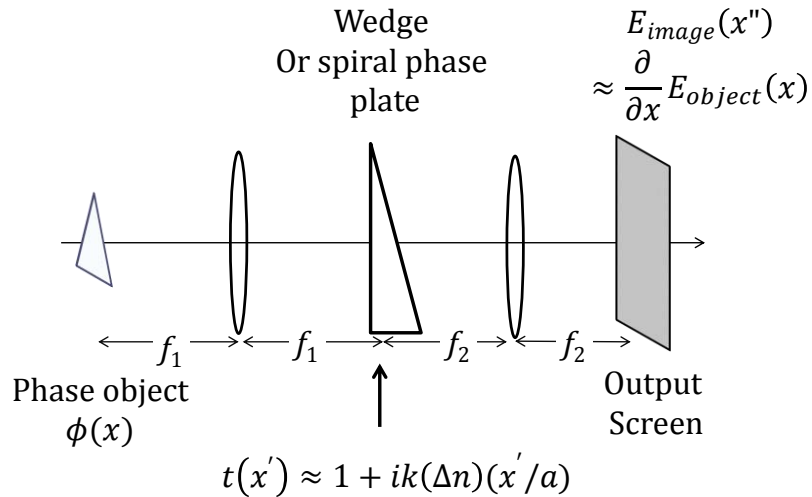


$$t(x, y) = \exp\left(i\frac{\pi}{2}\right) + ik(n-1)h(x, y) = i(1 + k(n-1)h(x, y)) \quad (21)$$

$$I(x, y) \propto |t(x, y)|^2 = 1 + 2k(n-1)h(x, y) + O(h^2) \quad (22)$$

Now the transmitted intensity reflects the phase information. Actually, since the intensity with phase change is nearly *linear* for small phase shifts, this method gives a direct image of the phase that is simple to interpret.

**b. Schlieren Method**



Schlieren (“streaks” in German) or shadowgraph imaging is important in the visualization of fluid flows, as it shows phase gradients of the object in a particular direction. To elaborate that effect, let’s model the transmission function of the phase mask (e.g. a glass wedge or spiral plate) as following:

$$AS(x', y') \approx 1 + ik(\Delta n)(x'/a) \quad (23)$$

The field transmitted through the fluid  $(x, y)$ , is illuminating on the aperture:

$$E_{object}(x, y) \propto \exp[i\phi(x, y)] \quad (24)$$

$$\begin{aligned}
 E_{image}(x'', y'') &\approx \mathcal{F}\left(AS\left(k_x \frac{f_1}{k}, k_y \frac{f_1}{k}\right) \times \mathcal{F}\left(E_{object}(x, y)\right)\right) \\
 E_{image}(x'', y'') &\approx \mathcal{F}\left(\left(1 + i(\Delta n)k_x \frac{f_1}{a}\right) \times \mathcal{F}\left(E_{object}(x, y)\right)\right) \\
 E_{image}(x'', y'') &\approx E_{object}(x, y) + (\Delta n) \frac{f_1}{a} \mathcal{F}\left(\mathcal{F}\left(\frac{\partial}{\partial x} E_{object}(x, y)\right)\right) \quad (25)
 \end{aligned}$$

$$E_{image}(x'', y'') \approx E_{object}(x, y) + (\Delta n) \frac{f_1}{a} \frac{\partial}{\partial x} E_{object}(x, y) \quad (26)$$

$$E_{image}(x'', y'') \approx E_{object}(x, y) \left[ 1 + i(\Delta n) \frac{f_1}{a} \frac{\partial}{\partial x} \phi(x, y) \right] \quad (27)$$

Note that using a mask with phase gradient, the intensity fringes of image are connected to the index gradient of the fluid flow! Such effect was first reported by Hooke and Huygens, when they used a candle to heat up the air in front of an observing lens. (see "Schlieren experiment 300 years ago", by J. RIENITZ, *Nature* **254**, 293 - 295 (27 March 1975); doi:10.1038/254293a0)

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