MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.717J/MAS.857J Optical Engineering **Spring '02**

Problem Set #1 Posted Feb. 6, 2002 — Due Wednesday Feb. 13, 2002

Notation: (u, v) are the spatial frequencies conjugate to the Cartesian coordinate pair (x, y). $\mathcal{H}(u, v)$ is the optical transfer function (OTF).

1. A unit amplitude, normally incident, monochromatic plane wave illuminates an object of maximum linear dimension D, situated immediately in front of a larger positive lens of focal length f (see Figure 1). Due to a positioning error, the intensity distribution is measured across a plane at a distance $f - \Delta$ behind the lens. How small must Δ be if the measured intensity distribution is to accuretely represent the Fraunhofer diffraction pattern of the object?

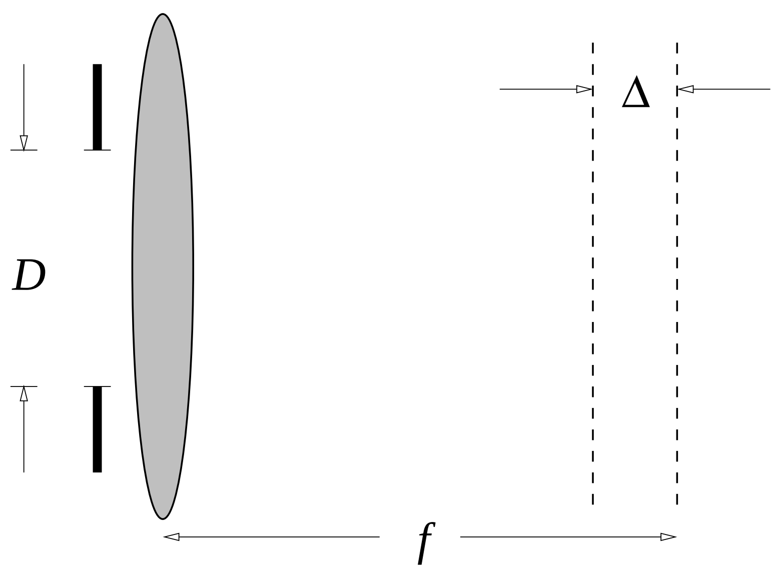

Figure 1

2. An infinite periodic square-wave grating with transmittivity as shown in Figure 2A is placed at the input of the optical system of Figure 2B. Both lenses are positive, $F/1$, and have focal length f. The grating is illuminated with monochromatic, spatially coherent light of wavelength λ and intensity I_0. The spatial period of the grating is $X = 4\lambda$. The element at the Fourier plane of the system is a nonlinear transparency with the intensity transmission function shown in Figure 2C, where the threshold and saturating intensities are $I_{\text{thr}} = I_{\text{sat}} = 0.1 I_0$.

2.a) To carry out the calculation analytically, you need to neglect the Airy patterns forming at the Fourier plane and pretend they are uniform bright dots. Explain why this assumption is justified and what effects it might have.

2.b) Derive and plot the intensity distribution at the output plane using the above assumption.

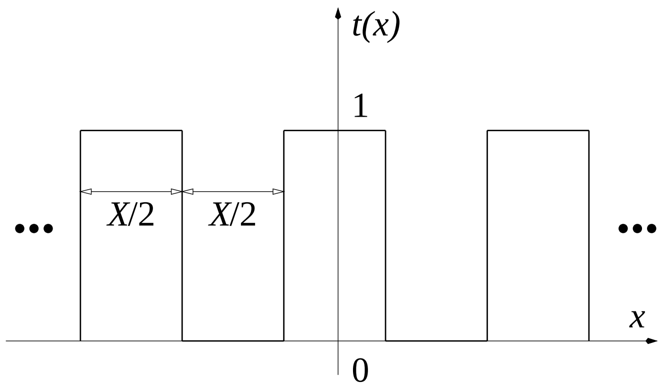

Figure 2A

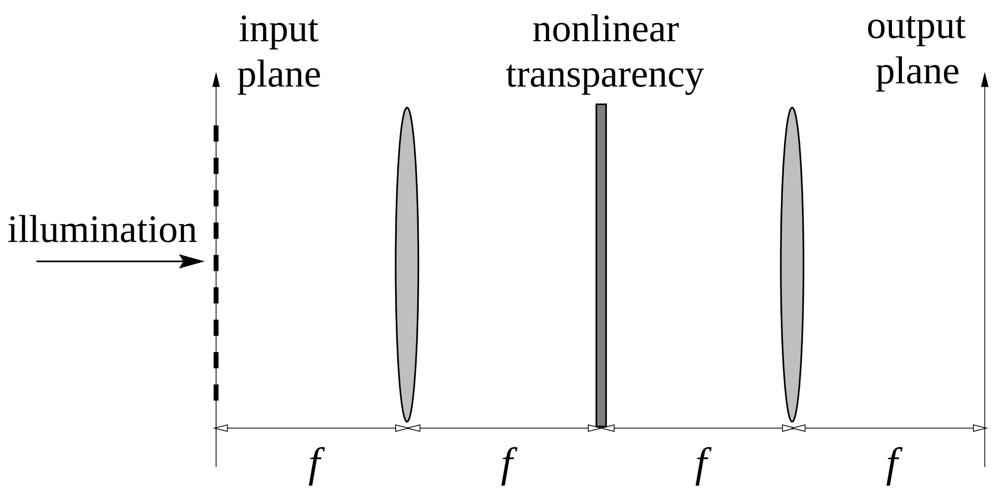

Figure 2B

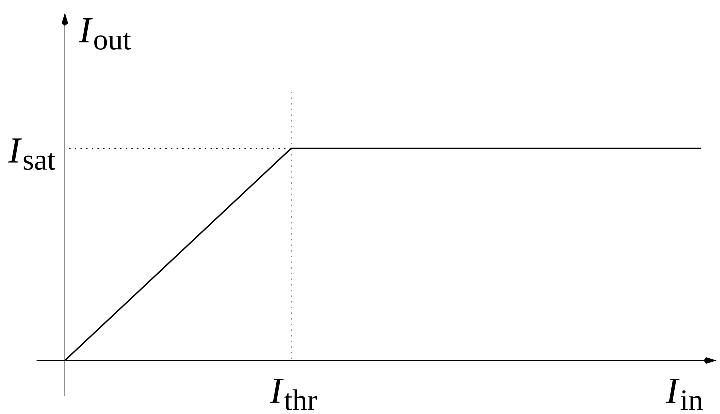

Figure 2C

3. A commonly cited quantity determining the seriousness of aberrations of an optical system is the Strehl number $\mathcal{D}$, which is defined as the ratio of the light intensity at the maximum of the point-spread function of the system with aberrations to that same maximum for that system in the absence of aberrations (*i.e.*, the diffraction-limited case; both maxima are assumed to exist on the optical axis).

3.a) Prove that $\mathcal{D}$ is equal to the normalized volume under the optical transfer function of the aberrated imaging system; that is, prove

$$\mathcal{D} = \frac{\iint_{-\infty}^{+\infty} \mathcal{H}_{\text{aberrated}}(u,v)\mathrm{d}u\mathrm{d}v}{\iint_{-\infty}^{+\infty} \mathcal{H}_{\text{diffr-lim}}(u,v)\mathrm{d}u\mathrm{d}v}$$

3.b) Argue that $\mathcal{D}$ is a real number and that $\mathcal{D} \leq 1$ always.

4. Sketch the u and v cross-sections of the optical transfer function of an incoherent imaging system having as a pupil function the two-pinhole combination shown in Figure 4. Assume $w < d$. Do *not* use MATLAB for this calculation. Explain briefly the appearance of your sketches, and be sure to label the various cutoff frequencies and center frequencies.

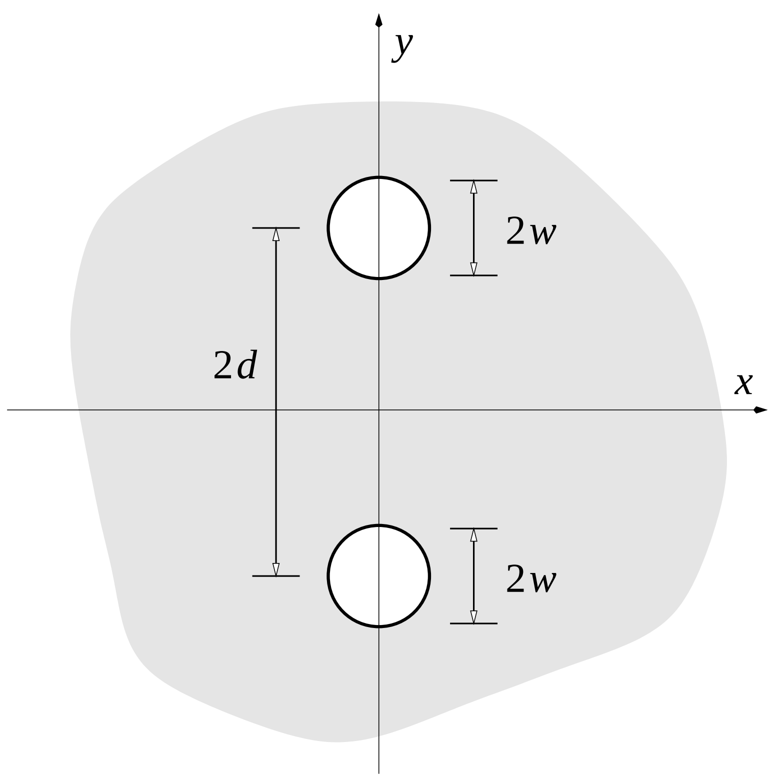

Figure 4

5. An ojbect with square-wave amplitude transmittance identical to the grating of Figure 2A is imaged by a lens with a circular pupil function. The focal length of the lens is 10 cm, the fundamental frequency of the square wave is $1/X$=100 cycles/mm, the object distance is 20 cm, and the wavelength is 1 μm. What is the minimum lens diameter that will yield *any variations* of intensity across the image plane for the cases of

5.a) Coherent object illumination?

5.b) Incoherent object illumination?

Hint The Fourier series expansion of the square wave of Figure 2A is

$$t(x) = \frac{1}{2} \sum_{n=-\infty}^{n=+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left\{i2\pi \frac{nx}{X}\right\},$$

where $\operatorname{sinc}(\xi) = \sin(\pi\xi)/(\pi\xi)$.