13.002
Introduction to Numerical Methods for Engineers
Problem set 2
Issued: Feb. 15, 2005
Due: Feb. 22, 2005

Problem 1.

The definite integral

$$y_n = \int_0^1 \frac{x^n}{x+5} dx, n = 0, 1, \ldots \infty$$

can be evaluated by recursion.

1. Show that the recurrence relation for the series is

$$y_n = \frac{1}{n} - 5y_{n-1}$$

2. Determine the starting value y_0

3. Show that $y_n > 0$ and that $y_n \to 0$ for $n \to \infty$.

4. Assume you have a (simple) computer using 3 digits after the decimal point. Calculate the results of the recursion up to $n = 4$. Discuss the results.

5. Make a computer program that uses the recursion to evaluate the integrals for $n = 0, \ldots 50$ and discuss your results.

6. Rewrite your program to use 'backward recurrence',

$$y_{50} = 0,$$
$$y_{n-1} = \frac{1}{5n} - \frac{y_n}{5}.$$

Discuss the results and speculate on what makes the difference.