Lecture 8: Cluster Model of QC

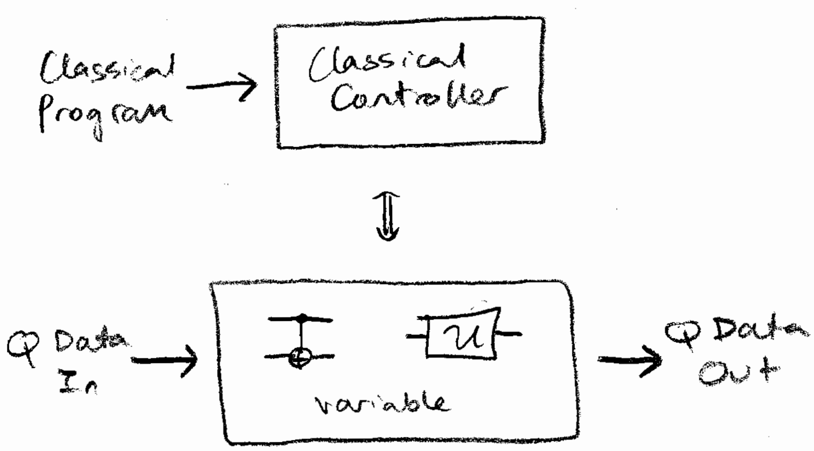
1) Models of QC
2) 1-bit teleportation
3) Cluster states
4) Cluster QC model
5) Fault tolerance

① Models of QC

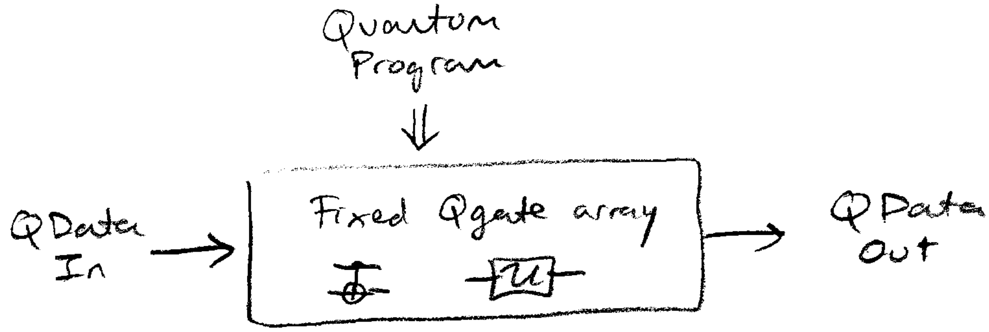
- Quantum circuit model

$$\langle \text{CNOT}, \ -\boxed{\mathcal{U}}- \rangle = \text{universal QC}$$

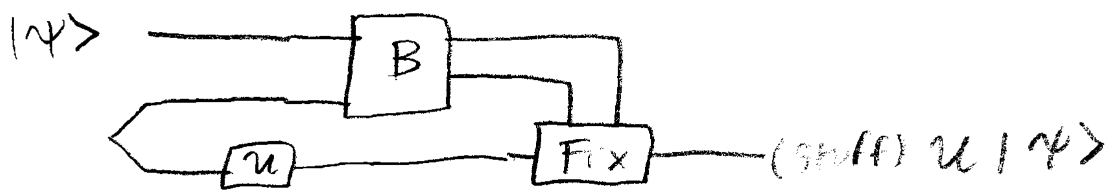
$$\langle \text{CNOT}, \ \mathcal{H}, \ T \rangle \approx \text{universal QC}$$



Different model:



Teleportation model



Cluster Model

1) Create state (perhaps complicated)
2) Measure qubits in various bases
   Feedback on measurement results

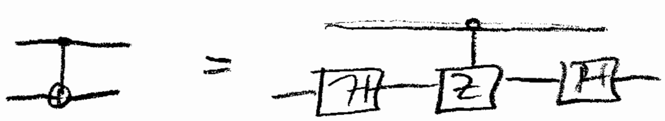
Model Resource Comparison

Model	States	Gates	Meas	Fault-tolerant
Q Circuit	$\lvert 0\rangle$	CNOT, $\boxed{U}$	$\lvert 0\rangle, \lvert 1\rangle$ basis	Not FT
Teleportation	$\lvert 0\rangle$ $\lvert \text{program}\rangle$	Clifford (incl. Pauli)	Bell	FT
Cluster	$\lvert \text{cluster}\rangle$	None	Arb. qubit	Not FT
Adiabatic QC			$\lvert 0\rangle, \lvert 1\rangle$	FT?

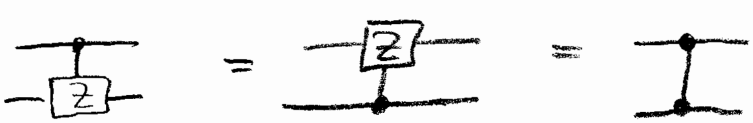
② 1-bit Teleportation

Swap:

CNOT:

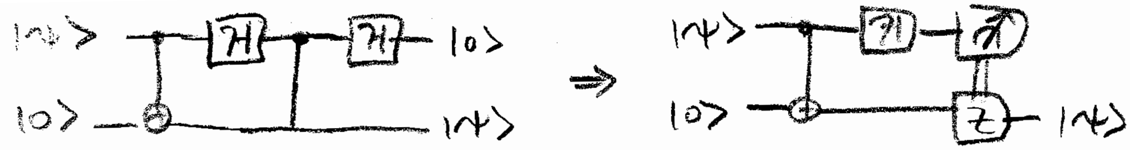


CPhase:



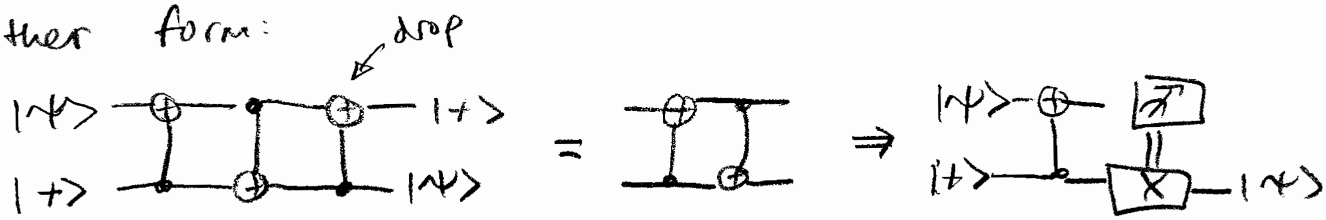
$\lvert 11\rangle \to -\lvert 11\rangle$

So SWAP



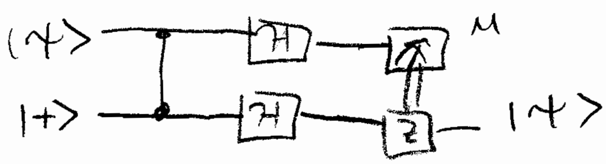
"Z-telep"

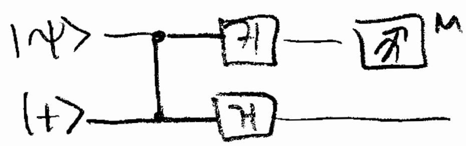
Another form:



"X-telep"

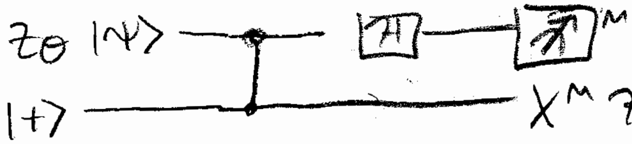
(Ike's favorite qcircuits!)



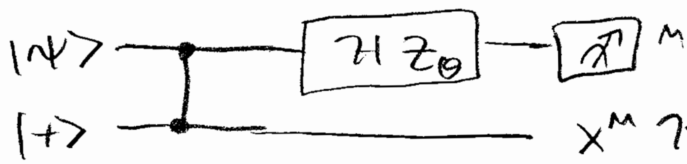


$\mathcal{H} Z^M |\psi\rangle = X^M \mathcal{H} |\psi\rangle$

More generally let $Z_\theta = R_z(\theta) = \exp(iZ\theta/2)$



$X^M \mathcal{H} Z_\theta |\psi\rangle$

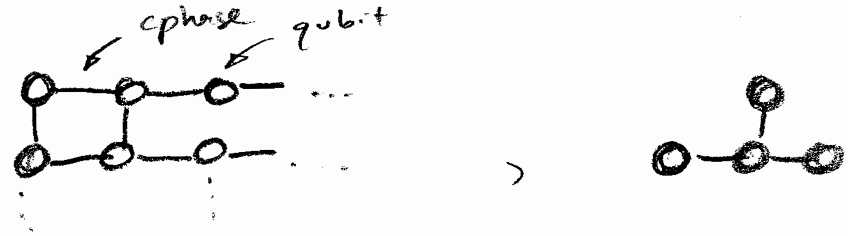


$X^M \mathcal{H} Z_\theta |\psi\rangle$

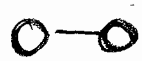
③ Cluster States

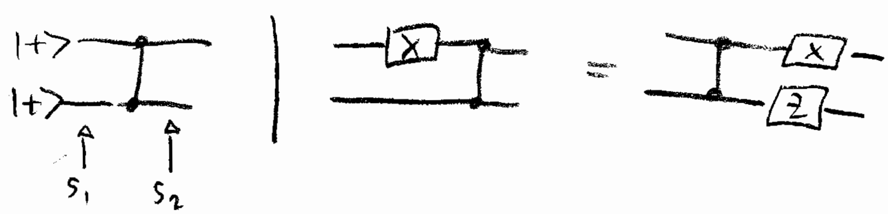
Def: A <u>cluster state</u> is a degree-1 graph with qubits as vertices & cphases as edges

e.g.



1) Initialize qubits in $|+\rangle$
2) Perform CPHASE btwn connected ~~nearest~~ neighbors
   – (Note: This is a stabilizer state!)

Ex:  =

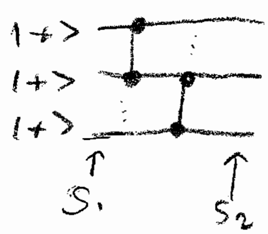


$S_1 = \langle XI, IX \rangle$

$S_2 = \langle XZ, ZX \rangle$

$|\psi_2\rangle = |00\rangle + |01\rangle + |10\rangle + |11\rangle$

Ex:  =



$S_1 = \langle XII, IXI, IIX \rangle$

$S_2 = \langle XZI, ZXZ, IZX \rangle$

$S_2 = \langle XZI,\ ZXZ,\ IZX \rangle$

Another form: $S_2 = \begin{array}{cc} X & Z \\ \left[\begin{matrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{matrix}\right| & \underbrace{\left.\begin{matrix} 0&1&0 \\ 1&0&1 \\ 0&1&0 \end{matrix}\right]}_{\text{adjacency Matrix}} \end{array} \begin{matrix} \leftarrow XZI \\ \leftarrow ZXZ \\ \leftarrow IZX \end{matrix}$

Y for (1 in X & 1 in Z)

<u>Def</u>: A <u>graph state</u> is a stabilizer state w/ state generators

$$\left[\ I \quad A\ \right]$$

I = identity

A = adjacency matrix of graph

<u>Fact</u>: $\{$Graph states$\} \subset \{$Stabilizer states$\}$

<u>Def</u>: <u>Local Clifford ops</u> $= \langle H, S \rangle$

<u>Def</u>: All stabilizer states are equiv to some graph states under LC ops

Ex: GHZ state $000 + 111$

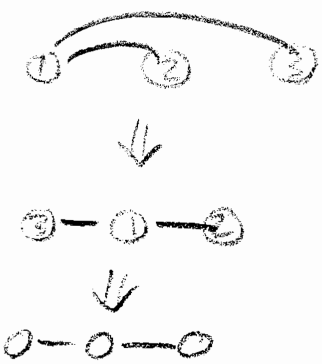
$$S = \langle ZZI, IZZ, XXX \rangle$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Can add rows to other rows
Can swap qubits within I or A

$$S = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{array}\right] \quad \text{\& do Gaussian elim}$$

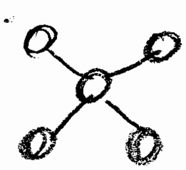
$$= \begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right] \\ \begin{array}{cccccc} & & & 1 & 2 & 3 \end{array} \end{array} \Rightarrow$$



Ex: $0^5 + 1^5$

$$S = \langle X^5, \underbrace{\begin{array}{c} ZZIII \\ IZZII \\ IIZZI \\ IIIZZ \end{array}}_{Z \to X} \rangle$$

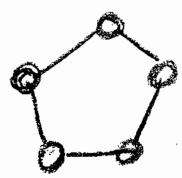
$$\begin{array}{c} XZZZZ \\ ZXIII \\ IXXII \\ IIXXI \\ IIIXX \end{array} \longrightarrow \begin{array}{c} XZZZZ \\ ZXIII \\ ZIXII \\ ZIIXI \\ ZIIIX \end{array} \longrightarrow$$



Ex: 5-qubit code

$$S = \left\langle \begin{array}{c} X\,Z\,Z\,X\,I \\ Z\,Z\,X\,I\,X \\ Z\,X\,I\,X\,Z \\ X\,I\,X\,Z\,Z \\ X\,X\,X\,X\,X \end{array} \right\rangle \quad \begin{array}{c} -1 \\ -2 \\ -3 \\ -4 \\ -5 \end{array}$$

$$\begin{array}{r c} & \begin{array}{ccccc|ccccc} & & X & & & & & Z & & \end{array} \\ \begin{array}{r} 523 \\ 215 \\ 4523 \\ 1523 \\ 433 \end{array} & \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{array} \right] \end{array}$$

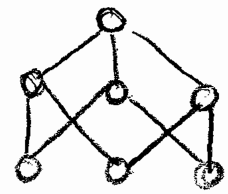


Every graph you can draw is a code
(though may correct for something silly)

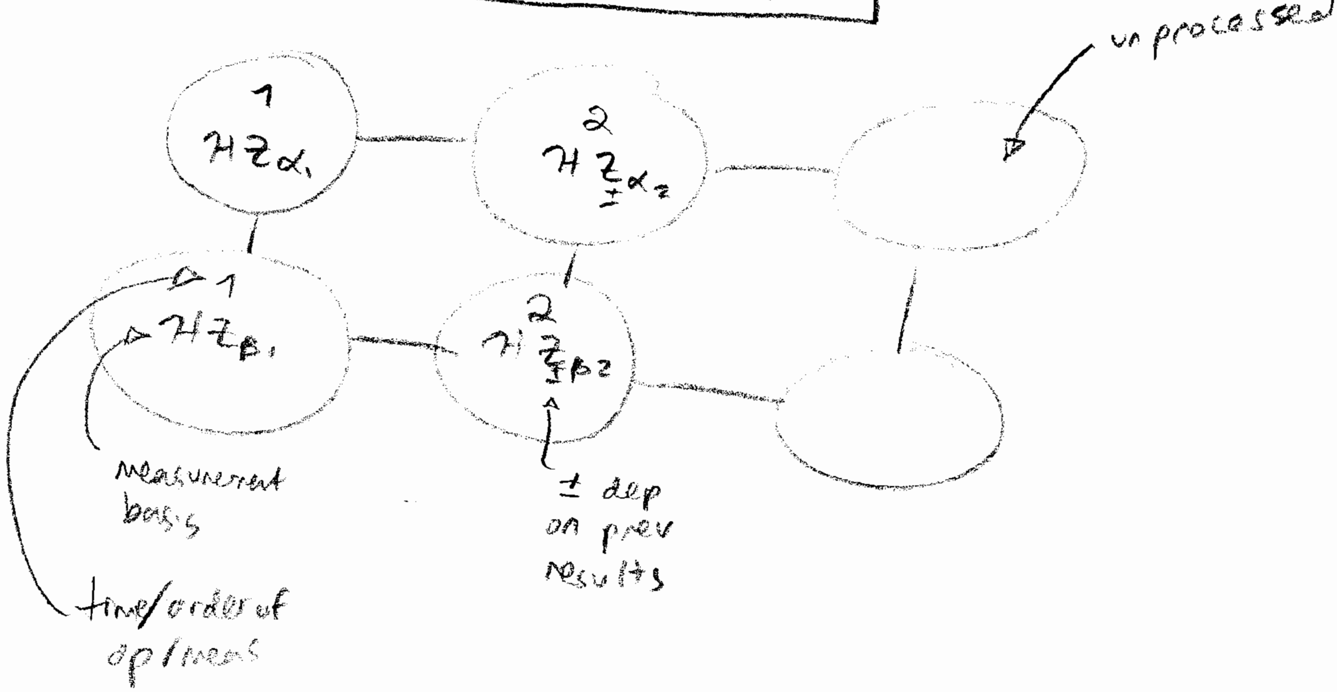
Hmmm: May exist more than 1 graph
(no "canonical graph" corr to a stabilizer)
(single)

Challenge: Define a canonical (single) graph state equiv to a given stabilizer state

Ex: [[7, 1, 3]] Steane code:



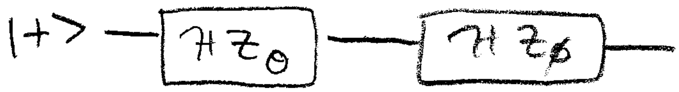
④ Cluster Model of QC



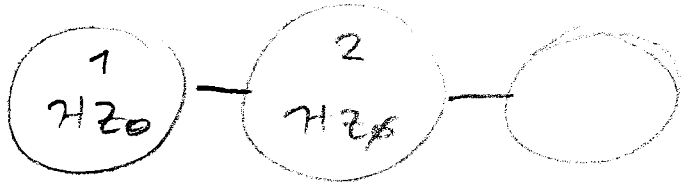
[Nielsen: quant-ph/0504097]

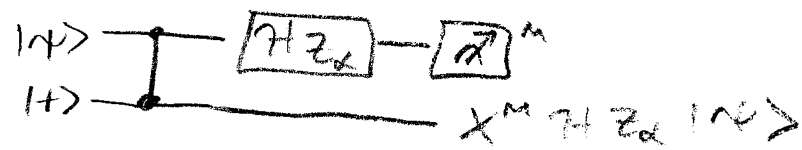
Meas Basis: $\boxed{\mathcal{H} Z_\alpha} \rightarrow \boxed{\vec{x}}$

Output: 1) Final unprocessed qubits
2) Meas recorded

Ex: 

$\Leftrightarrow \quad \mathcal{H} Z_\theta \mathcal{H} Z_\phi |+\rangle$

$\Leftrightarrow$ 

(Recall: 

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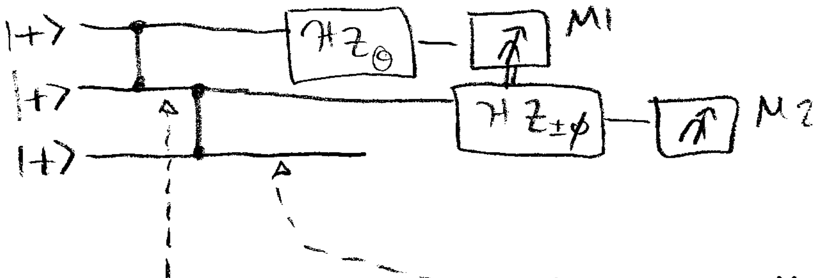


$X^{M_1} \mathcal{H} Z_\theta |+\rangle$

$X^{M_2} \mathcal{H}_{\pm\phi} X^{M_1} \mathcal{H} Z_\theta |+\rangle$

$\hookrightarrow X^{M_2} \mathcal{H} Z_{(-1)^{M}\phi} X^{M_1} \mathcal{H} Z_\theta |+\rangle$

$\hookrightarrow X^{M_2} \mathcal{H} X^{M_1} Z_\phi \mathcal{H} Z_\theta |+\rangle$

$\hookrightarrow \underbrace{X^{M_2} Z^{M_1}}_{\text{Know from meas results}} \underbrace{\mathcal{H} Z_\phi \mathcal{H} Z_\theta |+\rangle}_{\text{want this}}$

Recall $X Z_\theta X = Z_{-\theta}$

So choose $\pm$ sign dep on M:

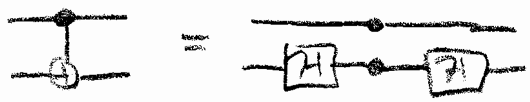
$M_1 = 1 \Rightarrow +$ sign

$M_1 = 0 \Rightarrow -$ sign

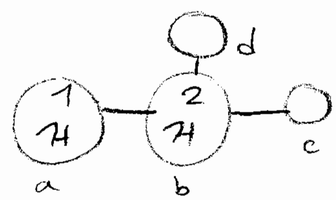
Pauli Frame

Can do qcomp if replace qubit → qubit + 2 cbits

Ex:



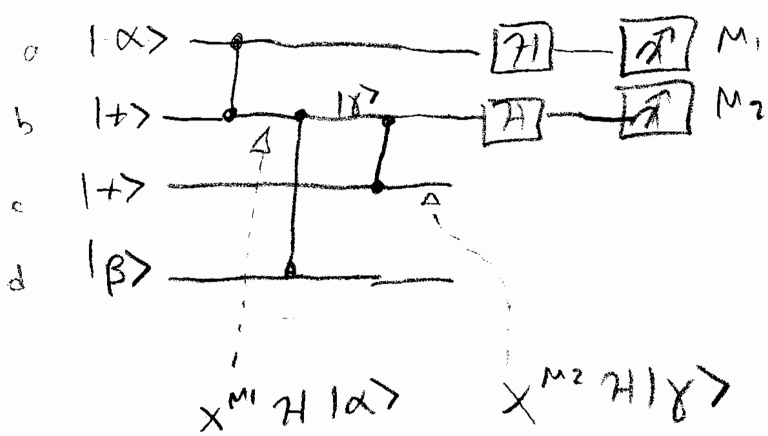
Use



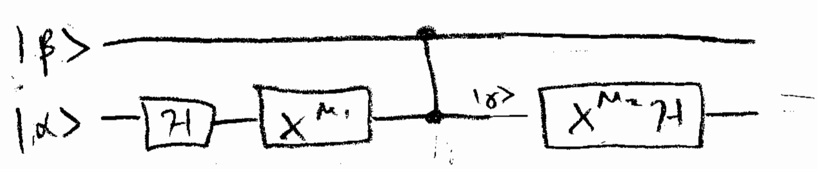
Claim:

Inputs: a, b

Outputs: CNOT [illegible] c, d



Altogether, equiv to:



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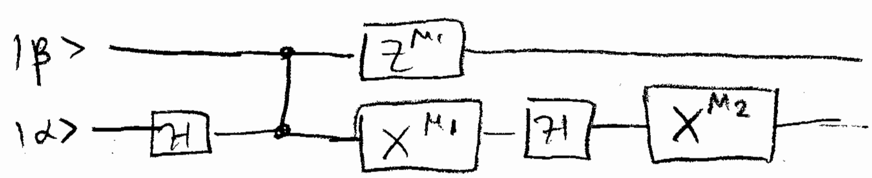

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