Operational Reactor Safety

22.091/22.903

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Professor of the Practice

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Lecture 2:
Reactor Physics Review

Topics to Be Covered

- Cross Sections
- Fission Process
- Infinite Reactor Systems
- Finite Reactor Systems
- Four Factor Formula
- Criticality Control
- Diffusion Theory
- Neutron Transport (Boltzman Equation)

Nuclear Reactor Physics Review

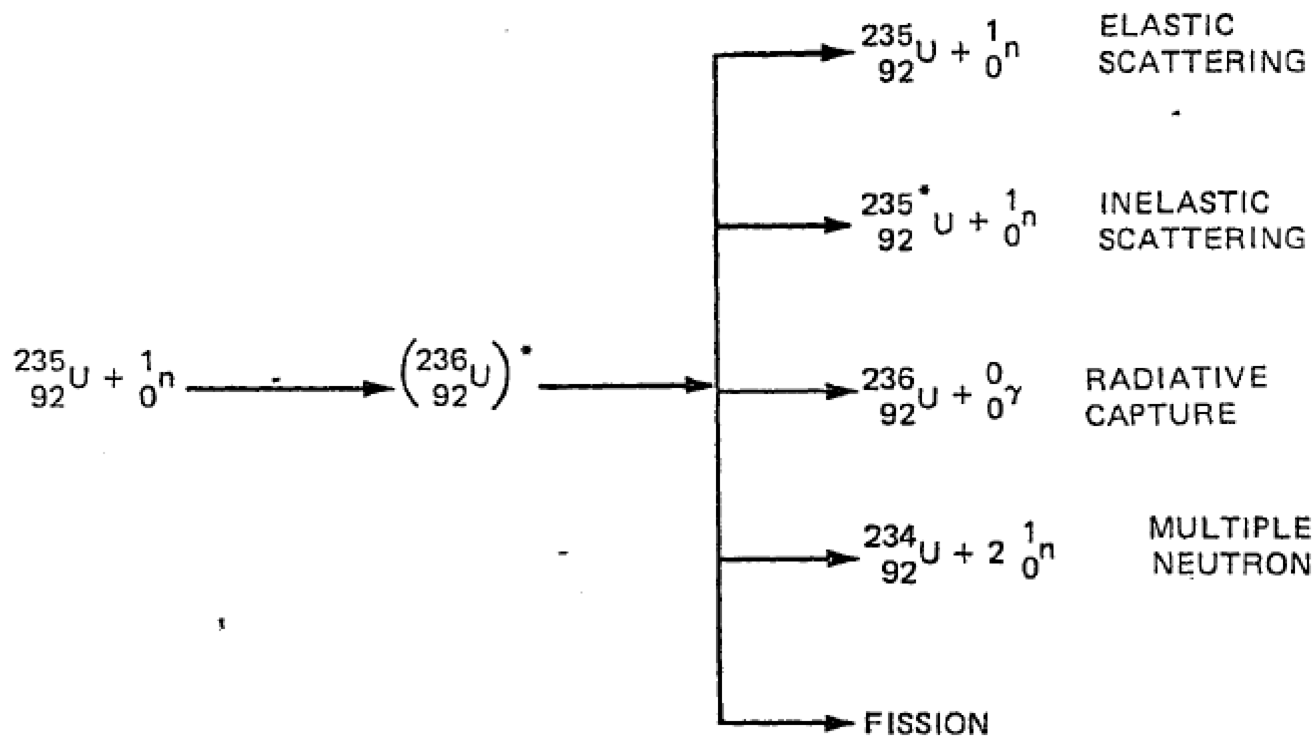


FIGURE 2-4
Possible outcomes from neutron irradiation of $^{235}_{92}U$.

Cross Sections

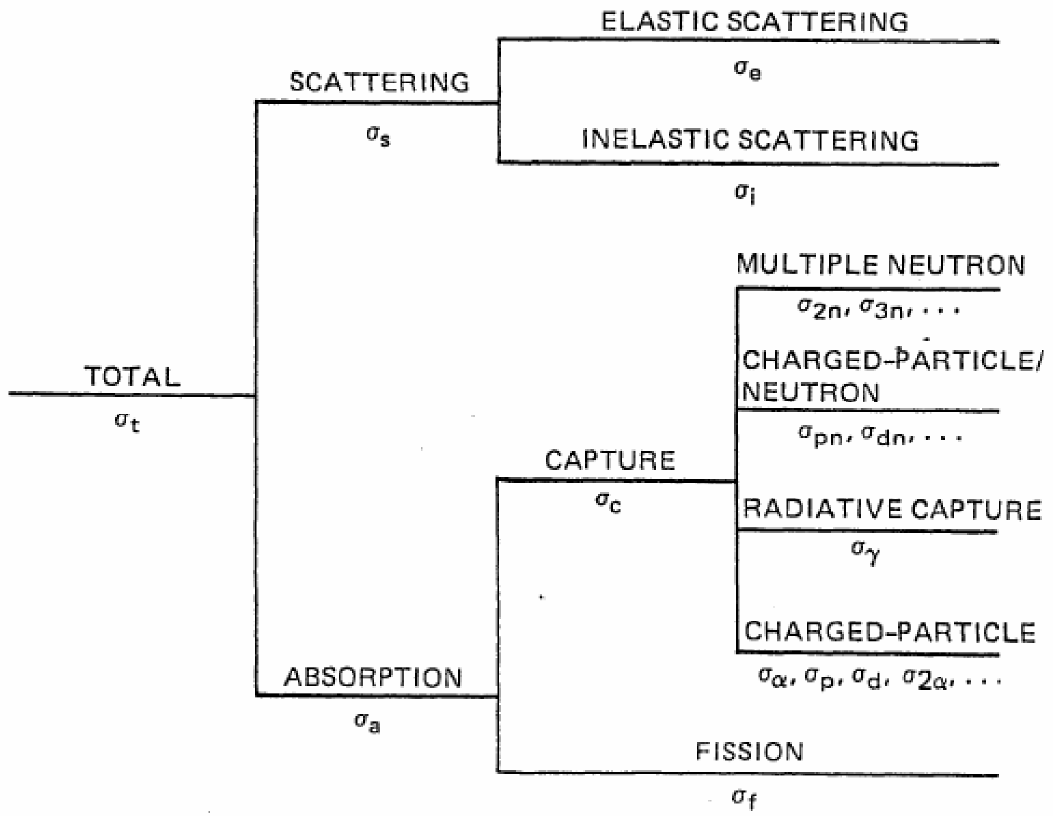


FIGURE 2-10
The hierarchy of microscopic cross sections used to calculate neutron interaction rates.

Cross Sections

TABLE 2-3
Thermal-Neutron Cross Sections and Parameters for Important Fissile and Fertile Nuclides

Nuclide	Cross section (b)† σ_a	σ_c	σ_f	α	ν‡	η
^{232}Th	7.4	7.4	—	—	—	—
^{233}U	577	46	531	0.087	2.50	2.30
^{235}U	584	99	585	0.169	2.44	2.09
^{238}U	2.68	2.68	—	—	—	—
^{239}Pu	1021	271	750	0.361	2.90	2.13
^{240}Pu	290	290	0.05	—	—	—
^{241}Pu	1371	361	1010	0.357	3.00	2.21

†Cross sections from Chart of the Nuclides (GE/Chart, 1989) for 0.025 eV "thermal" neutrons; σ_c has been assumed to be equal to the radiative-capture cross section σ_γ.

‡ν values at 0.025 eV from Hughes/BNL-325 (1958).

Uranium Cross Sections

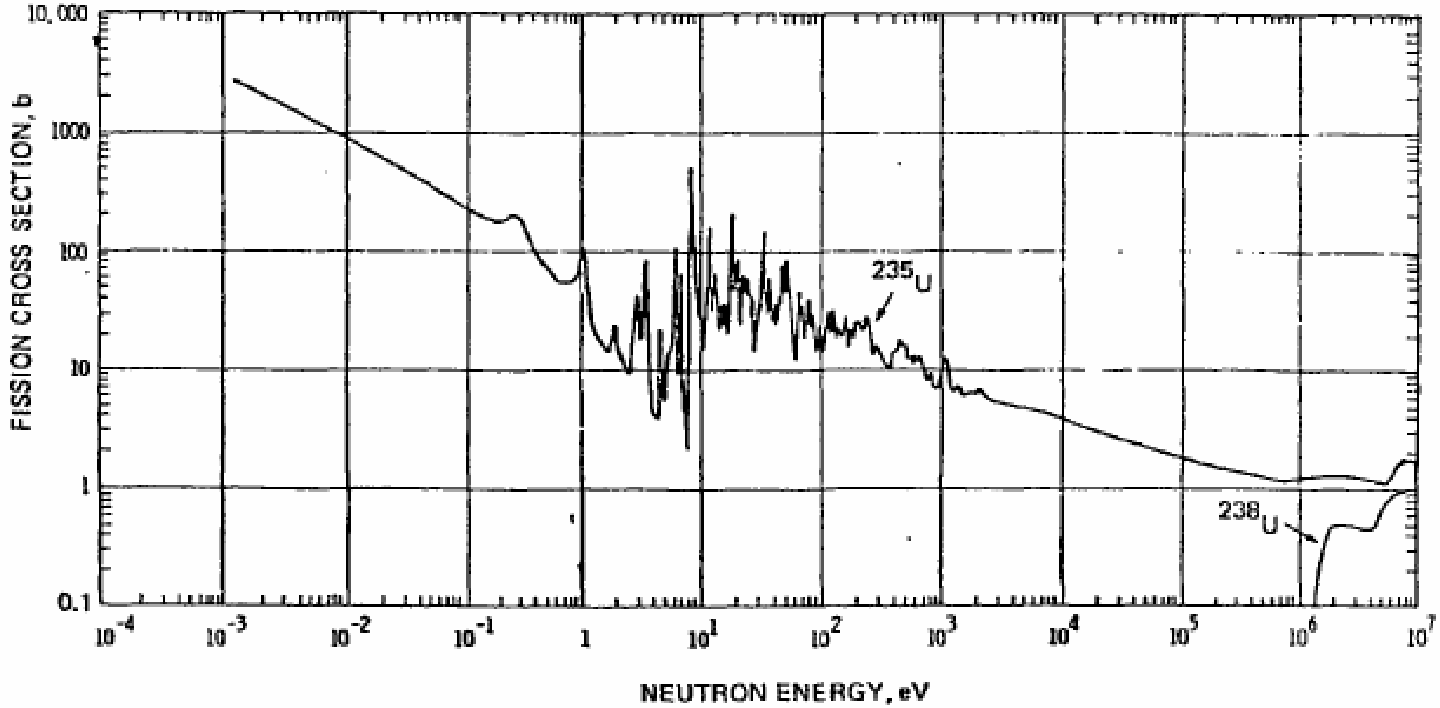


FIGURE 2-12

Microscopic fission cross section for fissile ^{235}U and fissionable ^{238}U. (Data from Hughes/BNL-325, 1955.)

Fission Chain Reaction

1. $n + {}^{235}_{92}U \rightarrow$ 2 Fission Products + 2.5 n + Other Stuff

2. 2.5 n
   - 1.5 Absorbed By Non-Fissioning Nuclei, or Escape From Reactor
   - $1 + {}^{235}_{92}U \rightarrow$ 2 Fission Products + 2.5 n + Other Stuff

3. 2.5 n
   - 1.5 Absorbed By Non-Fissioning Nuclei, or Escape From Reactor
   - $1 + {}^{235}_{92}U \rightarrow$ 2 Fission Products + 2.5 n + Other Stuff

Neutron Multiplication Factor	Condition	Consequence
$K > 1$	Supercritical	Neutron Population & Fission Rate Grow
$K = 1$	Critical	Neutron Population & Fission Rate Steady
$K < 1$	Subcritical	Neutron Population & Fission Rate Decrease

Fission Neutron Energy

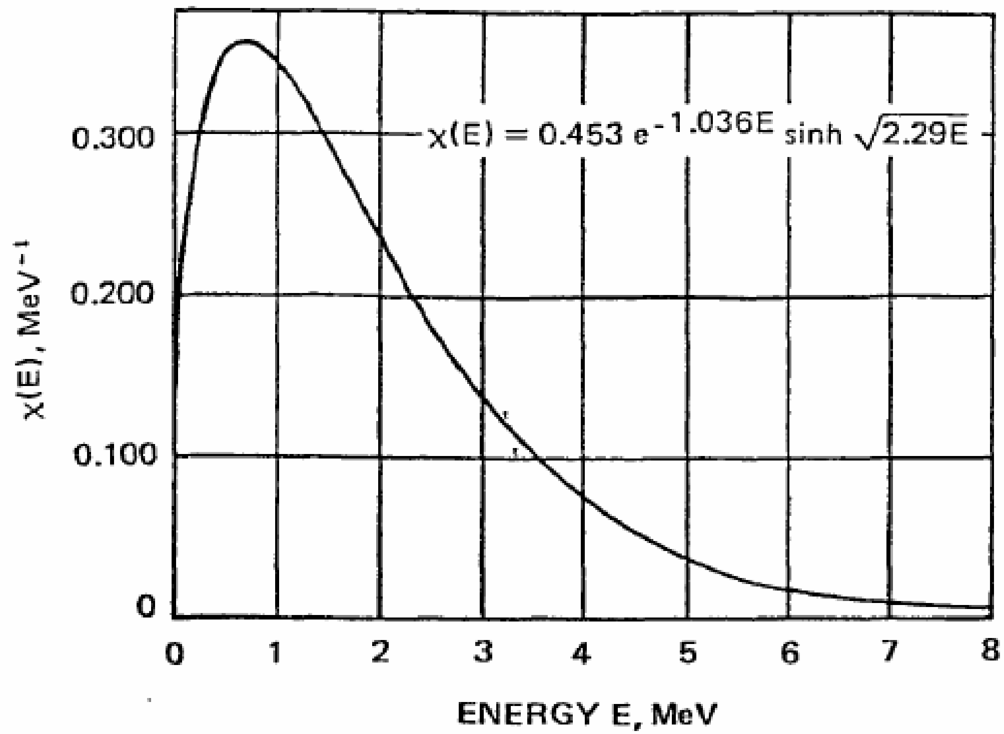


FIGURE 2-8
Fission-neutron energy spectrum $\chi(E)$ for the thermal fission of ^{235}U approximated by an empirical expression.

Fission Energy

TABLE 2-2
Representative Distribution of Fission Energy

Energy source	Fission energy (MeV)	Heat produced	
		MeV	% of tot:
Fission fragments	168	168	84
Neutrons	5	5	2.5
Prompt gamma rays	7	7	3.5
Delayed radiations			
Beta particles†	20	8	4
Gamma rays	7	7	3.5
Radiative capture gammas‡	—	5	2.5
Total	207	200	100

† Includes energy carried by beta particles and antineutrinos; the latter do not produce heat in reactc systems.

‡ Nonfission capture reactions contribute heat energy in all real systems; design-specific consideration may change this number by about a factor of two in either direction.

Fission Product Decay Chain

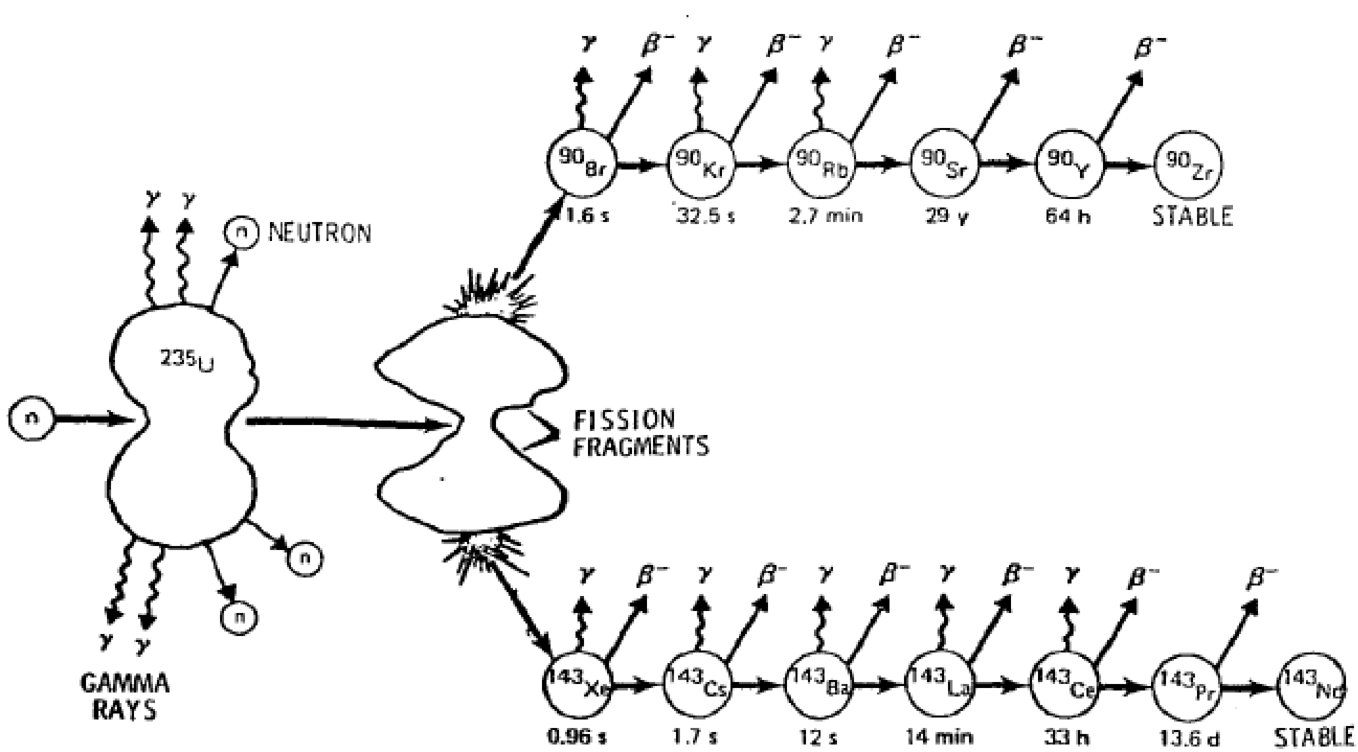


FIGURE 2-7

Two representative fission-product decay chains (from *different* fissions). (Adapted courtesy of U.S. Department of Energy.)

Fission Yield by Mass Number

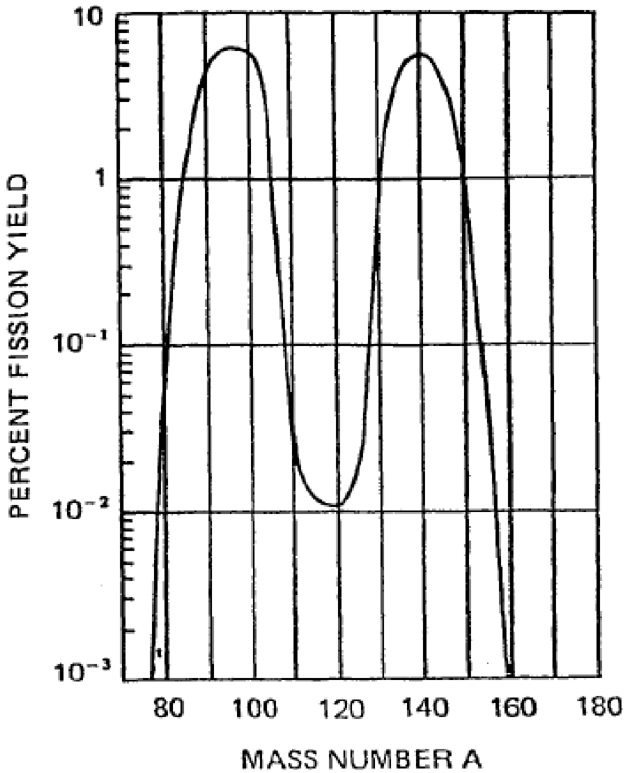


FIGURE 2-6

Fission yield as a function of mass number for thermal-neutron fission of ^{235}U. (Adapted from *Nuclear Chemical Engineering* by M. Benedict and T. H. Pigford, © 1957 by McGraw-Hill Book Company, Inc. Used by permission of McGraw-Hill Book Company.)

Chart of the Nuclides

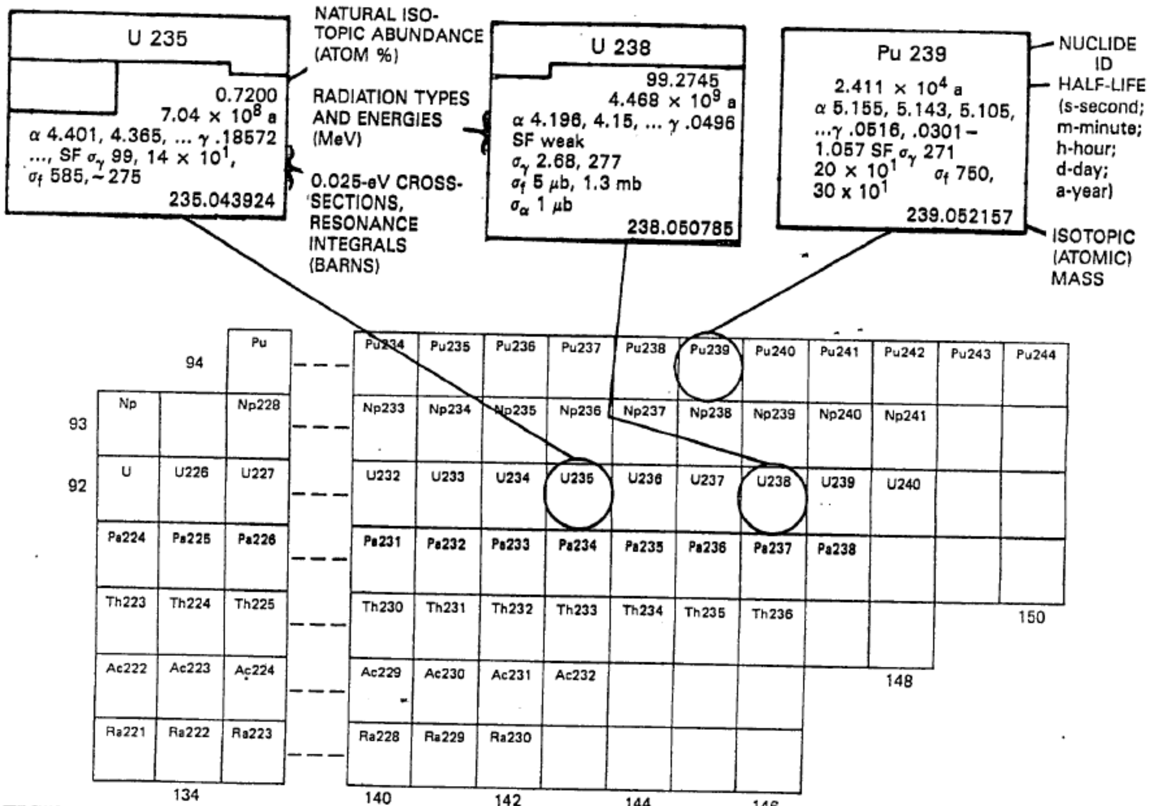


FIGURE 2-15

Structure and data presentation in the Chart of the Nuclides for ^{235}U, ^{238}U, and ^{239}Pu. (Adapted from the Chart of the Nuclides, courtesy of Knolls Atomic Power Laboratory, Schenectady, New York. Operated by the General Electric Company for the United States Department of Energy Naval Reactors Branch.)

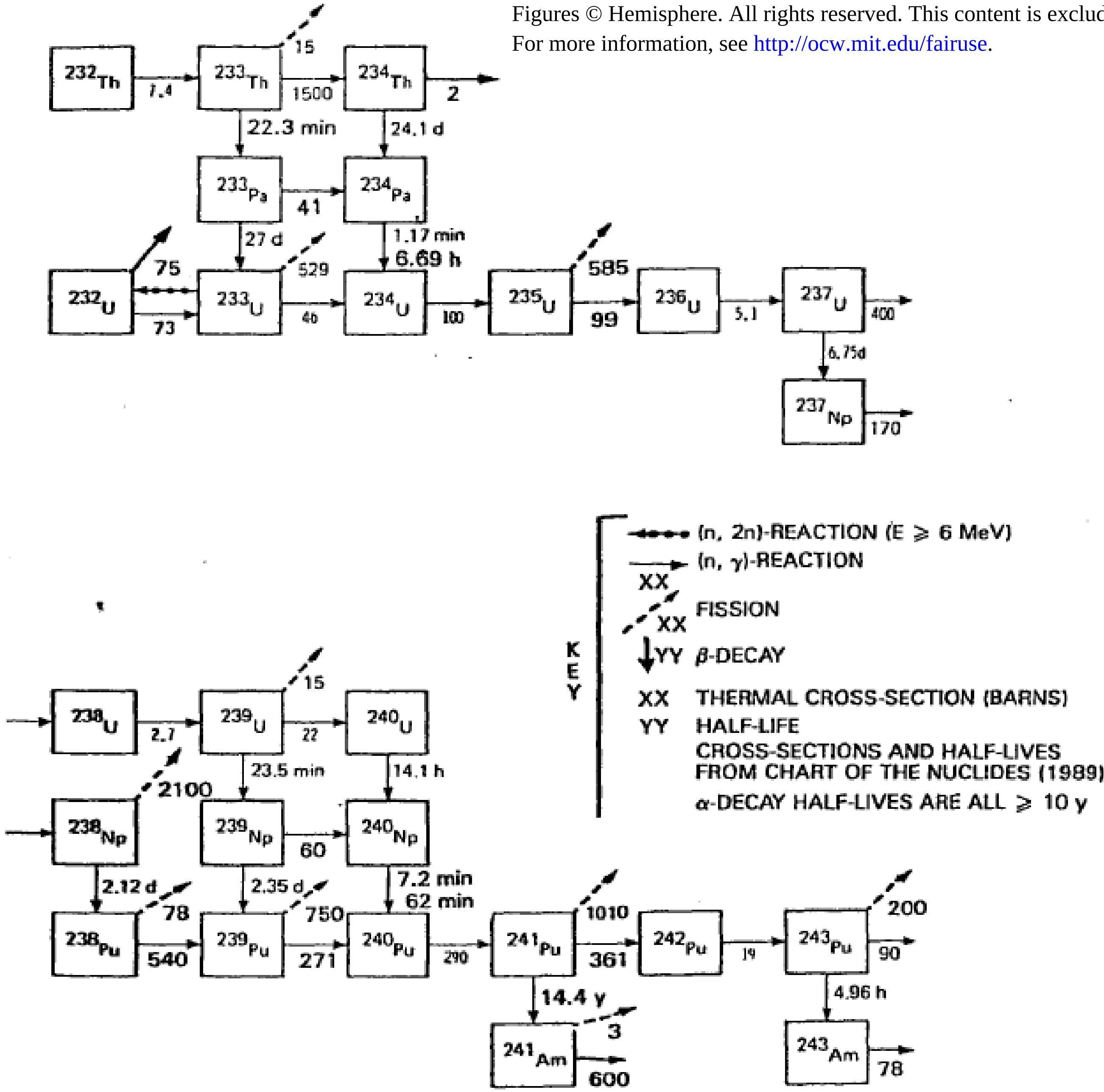


FIGURE 2-16
Neutron irradiation chains for heavy elements of interest for nuclear reactors.

Conversion Chain: Fertile→ Fissile

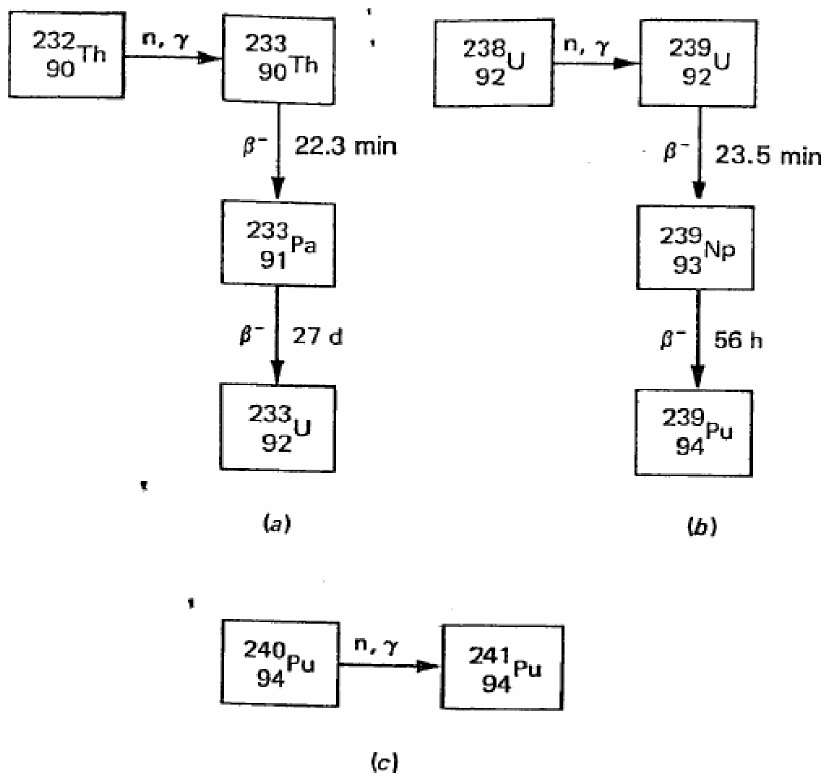
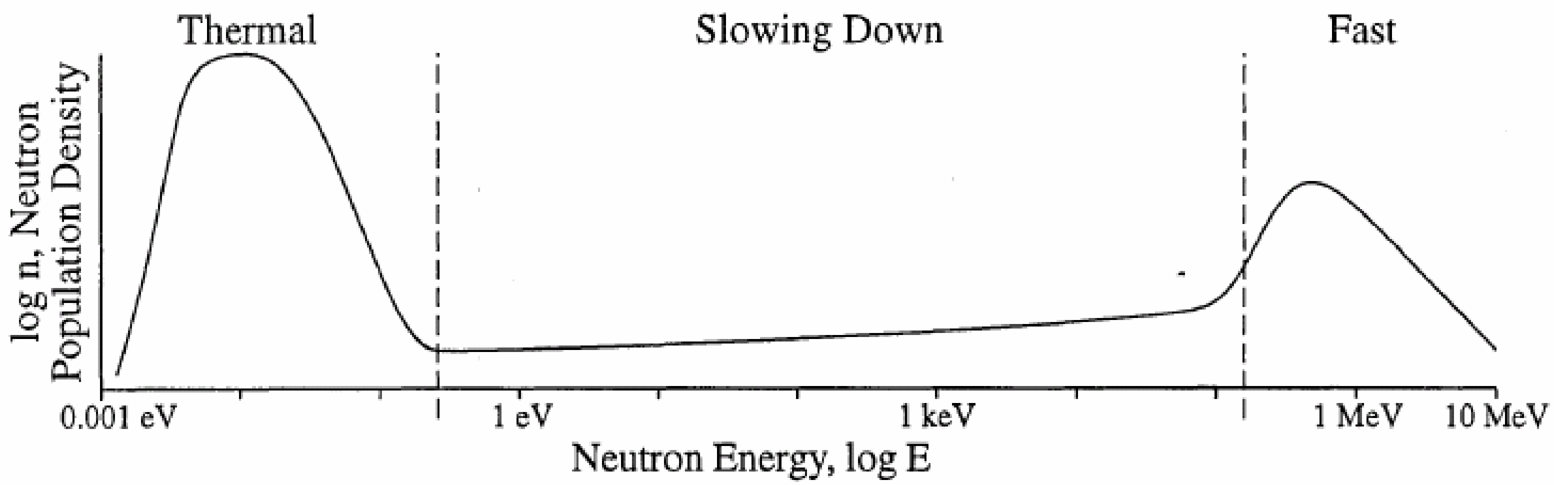


FIGURE 2-5
Chains for conversion of fertile nuclides to fissile nuclides: (*a*) $^{232}_{90}$Th to $^{233}_{92}$U; (*b*) $^{238}_{92}$U to $^{239}_{94}$Pu; (*c*) $^{240}_{94}$Pu to $^{241}_{94}$Pu.

Reaction Rates*

- How do we make power?
- Factors to consider in design
  - Changes in fuel material
  - Life of reactor core
  - Reactivity swing
  - Refueling strategies

NEUTRON SLOWING DOWN

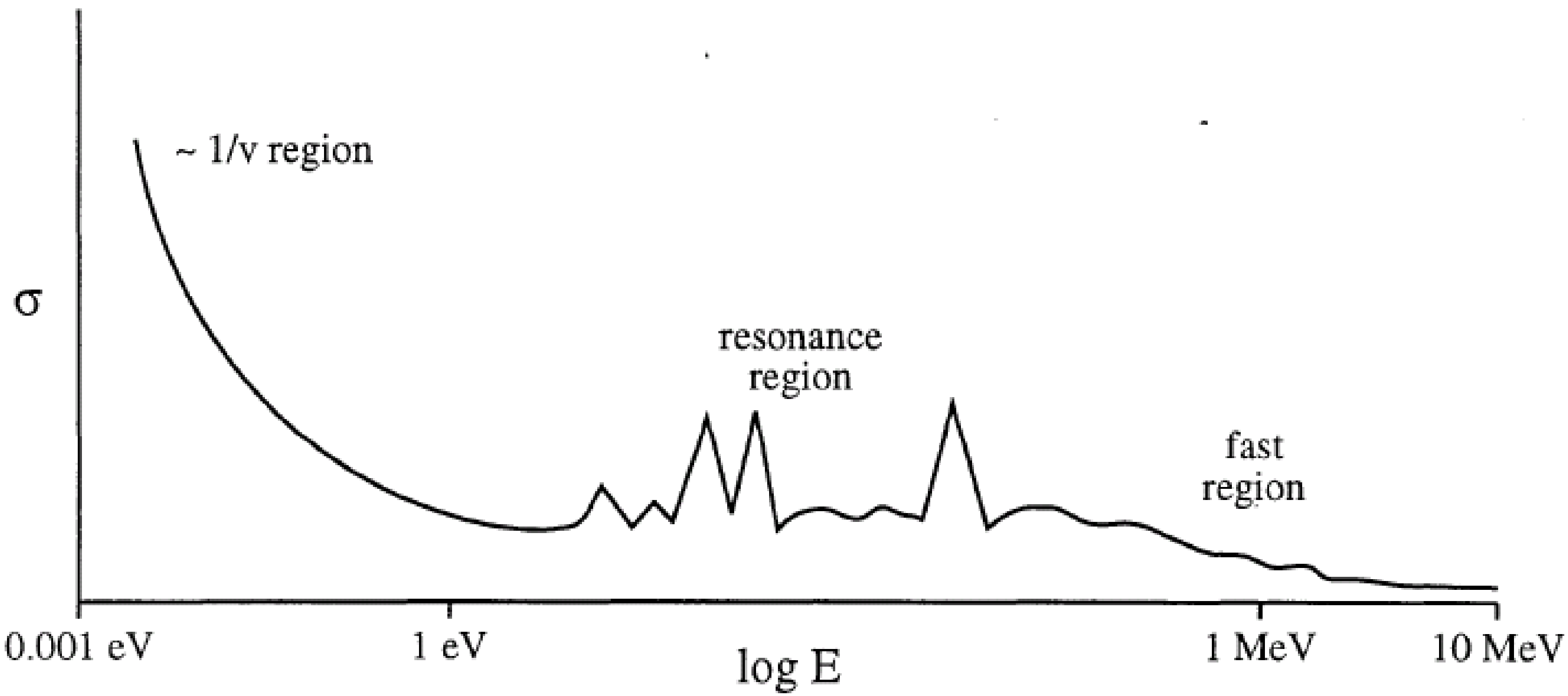


Fast: 1–10 MeV, Zone of Neutron Birth From Fission

Slowing Down: 1 MeV–1 eV, Zone of Neutron Collisions With Moderator Nuclei, and Kinetic Energy Exchange to Target Nuclear; Zone of Resonance Absorption

Thermal: < 1 eV, Zone of Neutron Elimination (via Absorption or Escape); Zone of Kinetic Energy Exchange To, From Neutron and Target Nucleus; Rate Depends Upon Temperature of Target Medium

ENERGY DEPENDENCE OF TYPCIAL TARGET NUCLEUS NEUTRON REACTOR CROSS SECTION



Neutron Moderation Parameters

TABLE 4-1
Parameters Related to Neutron Moderation by Nuclides of Interest in Power Reactors

Nuclide	$\frac{\Delta E_{max}}{E}$	$\alpha = \frac{E_{min}}{E}$	$(\Delta u)_{max}$†	ξ	Number of collisions‡
$^{1}_{1}H$	1.000	0.000	–	1.000	18
$^{2}_{1}D$	0.889	0.111	2.198	0.725	24
$^{12}_{6}C$	0.284	0.716	0.334	0.158	111
$^{23}_{11}Na$	0.160	0.840	0.174	0.085	206
$^{238}_{92}U$	0.017	0.983	0.017	0.0084	2084

†Lethargy change associated with maximum energy change.
‡Average requirement for a 1-MeV fission neutron to be reduced in energy to 0.025 eV.

Infinite & Finite Reactor Systems*

- Infinite Systems
  - Neutron Multiplication
  - Four Factor Formula
- Finite Systems
  - Leakage
  - Six Factor Formula
- Diffusion Theory

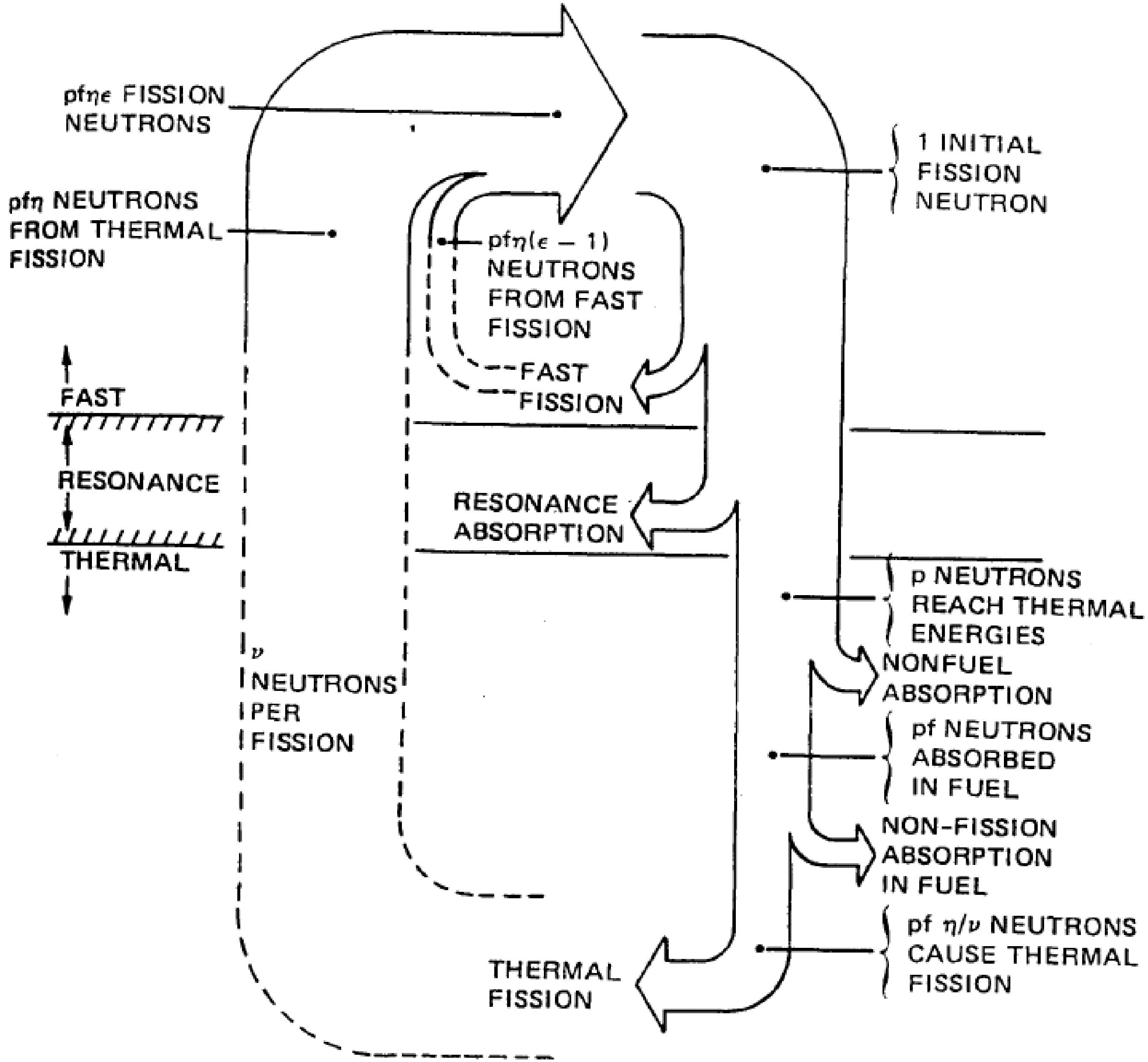
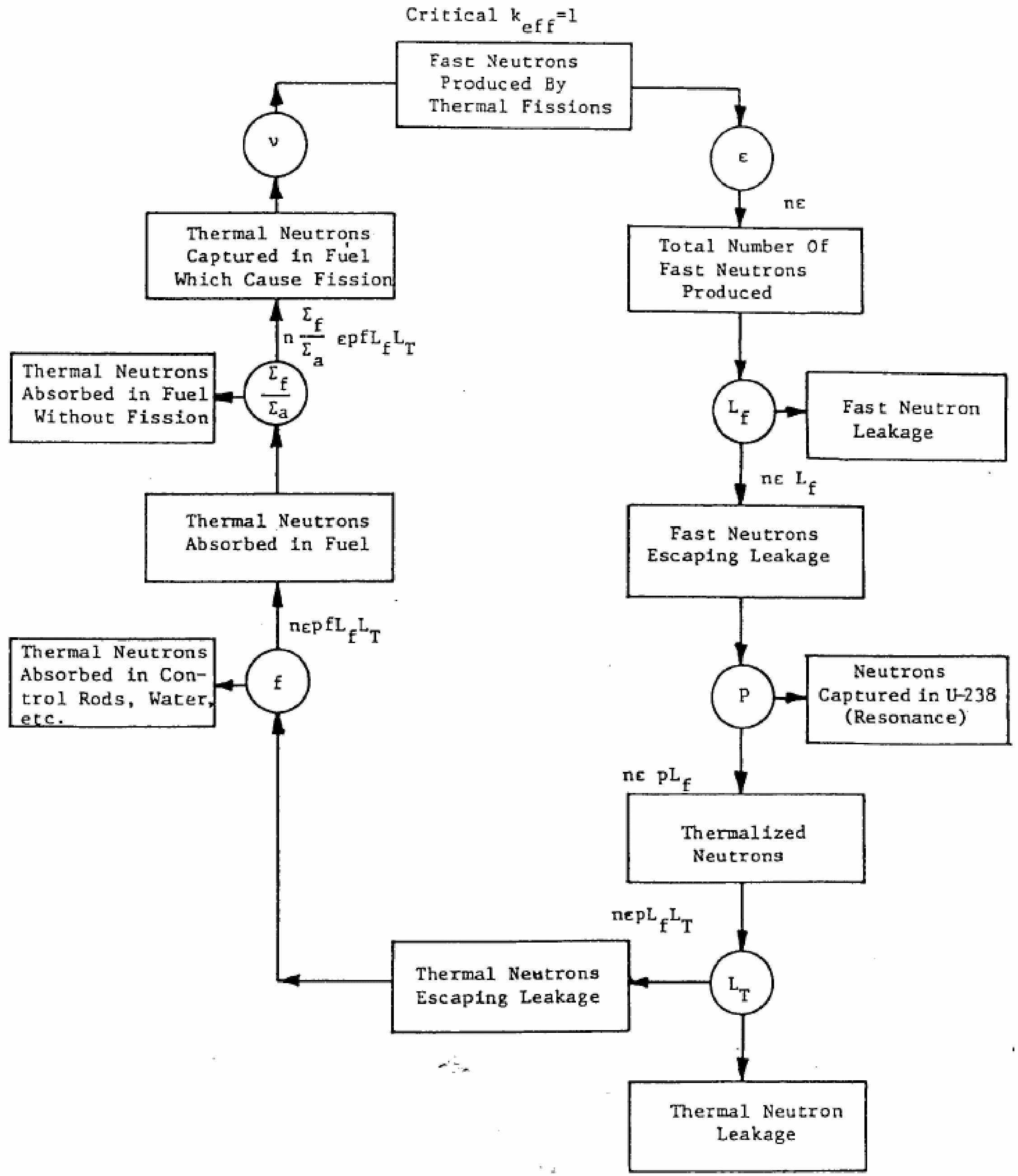


FIGURE 4-3

Flow diagram for neutrons in a thermal reactor from the standpoint of the four-factor formula.



Neutron Life Cycle

Core Multiplication Factor

1. It is useful to define a 'core multiplication factor' which is denoted by the symbol 'K' and which is the product of the six factors that define the neutron life cycle. Thus,

$$K = \varepsilon L_f p L_t f \eta$$

2. The above expression, which is called the 'six-factor formula, has physical meaning:

$$K = \frac{\text{Neutrons Produced from Fission}}{\text{Neutrons Absorbed + Neutron Leakage}} \quad \text{or}$$

$$K = \frac{\text{Number Neutrons in Present Generation}}{\text{Number Neutrons in Preceding Generation}}$$

$$K = \frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_3}{n_2}$$ when n is the number of neutrons in each generation.

3. If K is unity, the reactor is critical.

4. If we know the K-value for a reactor core, we can determine the rate of change of its neutron population. This is most useful in reactor startups.

Definitions of Neutron Life Cycle Factors

$$\varepsilon = \frac{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}{\text{Number of Fast Neutrons Produced from Thermal Fission}}$$

$$L_f = \frac{\text{Total Number of Fast Neutrons Escaping Leakage}}{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}$$

$$p = \frac{\text{Total Number of Thermalized Neutrons}}{\text{Total Number of Fast Neutrons Escaping Leakage}}$$

$$L_t = \frac{\text{Total Number of Thermal Neutrons Escaping Leakage}}{\text{Total Number of Thermalized Neutrons}}$$

Definitions of Neutron Life Cycle Factors (cont.)

$$f = \frac{\text{Thermal Neutrons Absorbed in Fuel}}{\text{Total Number of Thermal Neutrons Escaping Leakage}}$$

$$\eta = \frac{\text{Thermal Neutrons Captured in Fuel Which Cause Fission}}{\text{Thermal Neutrons Absorbed in Fuel}} \quad \text{or}$$

$$\eta = \frac{\text{Number of Fast Neutrons Produced from Thermal Fission}}{\text{Thermal Neutrons Absorbed in the Fuel}}$$

Of the above factors, the reactor operator can alter 'f' by changing the control rod position or by adjusting the soluble poison content. The leakage terms also vary during routine operation whenever coolant temperature changes. The other terms are fixed by the fuel type.

NEUTRON LOSS MECHANISMS

- Escape From Reactor
  - Reactor length scale: $L \sim m$
  - Neutron length scale: $\lambda \sim 0.1$ m
  - Probability of Escape Grows as (λ/L) Grows
- Capture By Reactor Materials
  - Control materials

    Control rods (e.g., B, Cd)

    Burnable poisons (e.g., Gd, Sm)
  - Structural materials (Fe, Zr)
  - Moderator, used to slow down neutrons (H, C)
  - Coolant, used to remove heat from reactor (H_2O, He, Na)

Criticality Control

- K should = 1 for 18 to 24 months
  - How ?
    - Fuel excess reactivity (fuel)
    - Balance with control rods or soluble boron
    - Burnable poisons in fuel to deplete with time
    - Leakage
    - On-line refueling – CANDU – Pebble Beds

Diffusion Theory*

- One group
  - Neutron Balance for critical reactor
  - Consider production absorption and leakage
- Multi-group
  - Include different energy groups

Diffusion Theory Fluxes and Bucklings

TABLE 4-2
Diffusion Theory Fluxes and Bucklings for Bare Critical Systems of Uniform Composition

Geometry	Dimensions	Normalized flux $\frac{\Phi(r)}{\Phi(0)}$	Geometric buckling B_g^2
Sphere	Radius R	$\frac{1}{r}\sin\left(\frac{\pi r}{R}\right)$	$\left(\frac{\pi}{R}\right)^2$
Finite cylinder	Radius R, height H (centered about $z = 0$ and extending to $z = \pm H/2$)	$J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$
Infinite cylinder	Radius R	$J_0\left(\frac{2.405r}{R}\right)$	$\left(\frac{2.405}{R}\right)^2$
Rectangular parallelepiped [cuboid]†	$A \times B \times C$ (centered about $x = y = z = 0$ and extending to $x = \pm A/2$, etc.)	$\cos\left(\frac{\pi x}{A}\right)\cos\left(\frac{\pi y}{B}\right)\cos\left(\frac{\pi z}{C}\right)$	$\left(\frac{\pi}{A}\right)^2 + \left(\frac{\pi}{B}\right)^2 + \left(\frac{\pi}{C}\right)^2$
Infinite slab	Thickness A (centered about $x = 0$ and extending to $x = \pm A/2$)	$\cos\left(\frac{\pi x}{A}\right)$	$\left(\frac{\pi}{A}\right)^2$

†The term *cuboid*—a synonym for rectangular parallelepiped used in the KENO Monte Carlo code—is commonly employed in the field of nuclear criticality safety (and may catch on elsewhere).

Multigroup Calculations

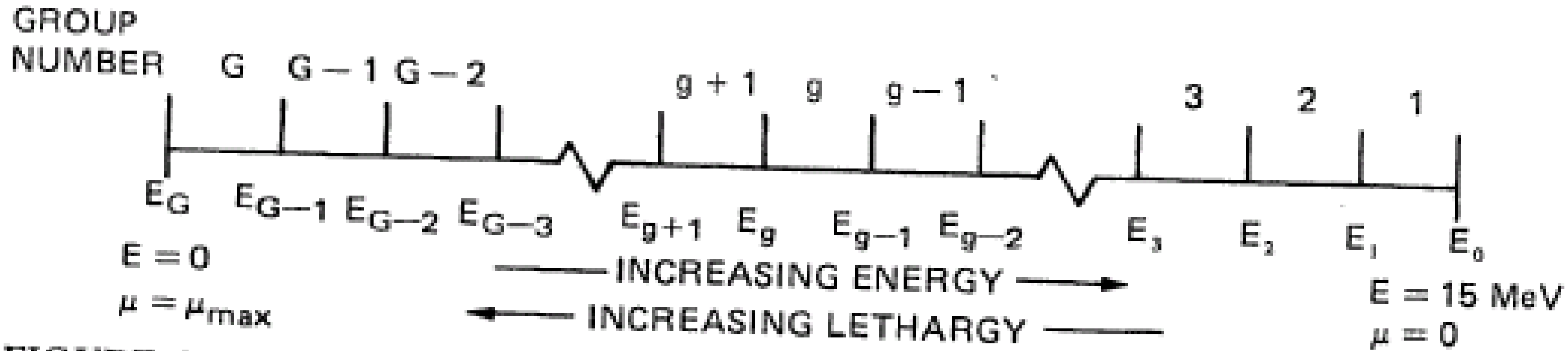


FIGURE 4-6
Partitioning of the energy axis for multigroup calculations.

Diffusion Theory Flux Shapes

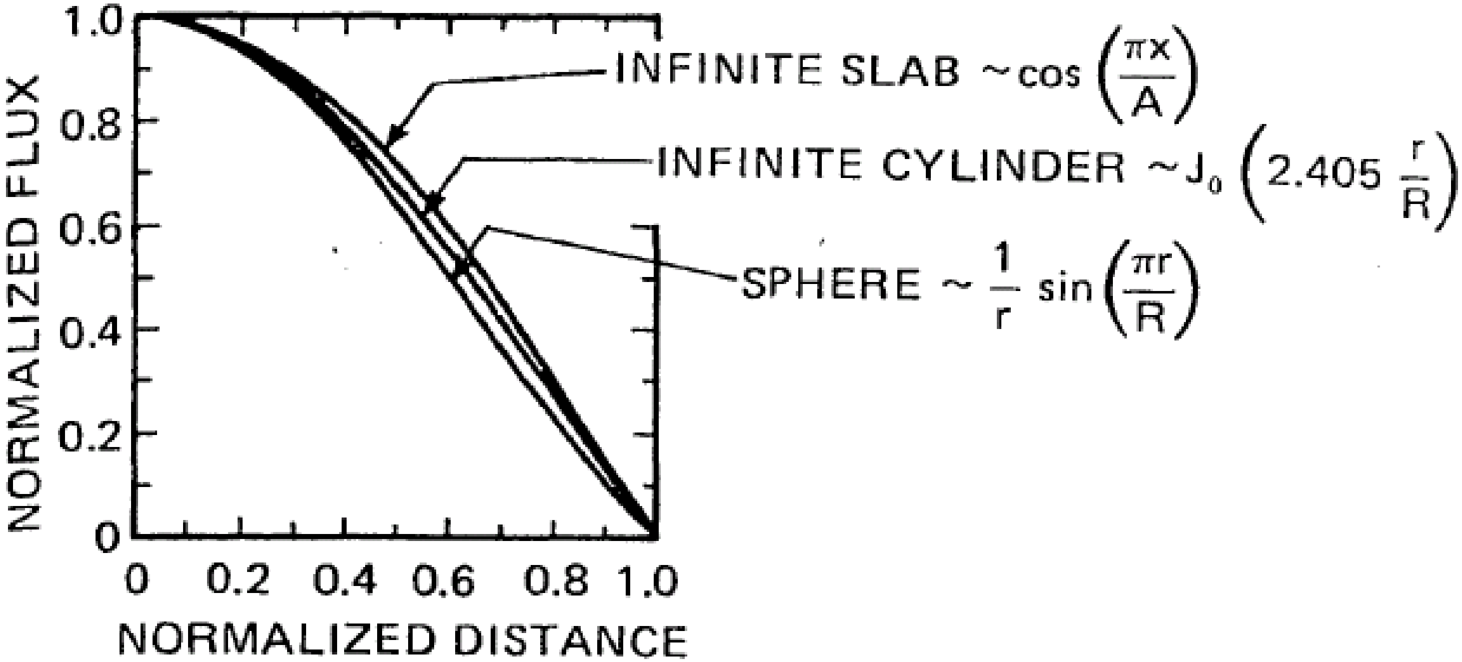


FIGURE 4-5

Normalized flux shapes predicted by diffusion theory for critical spheres of ra
cylinders of radius R, and infinite-area slabs of thickness A.

Fast and Thermal Flux Shapes

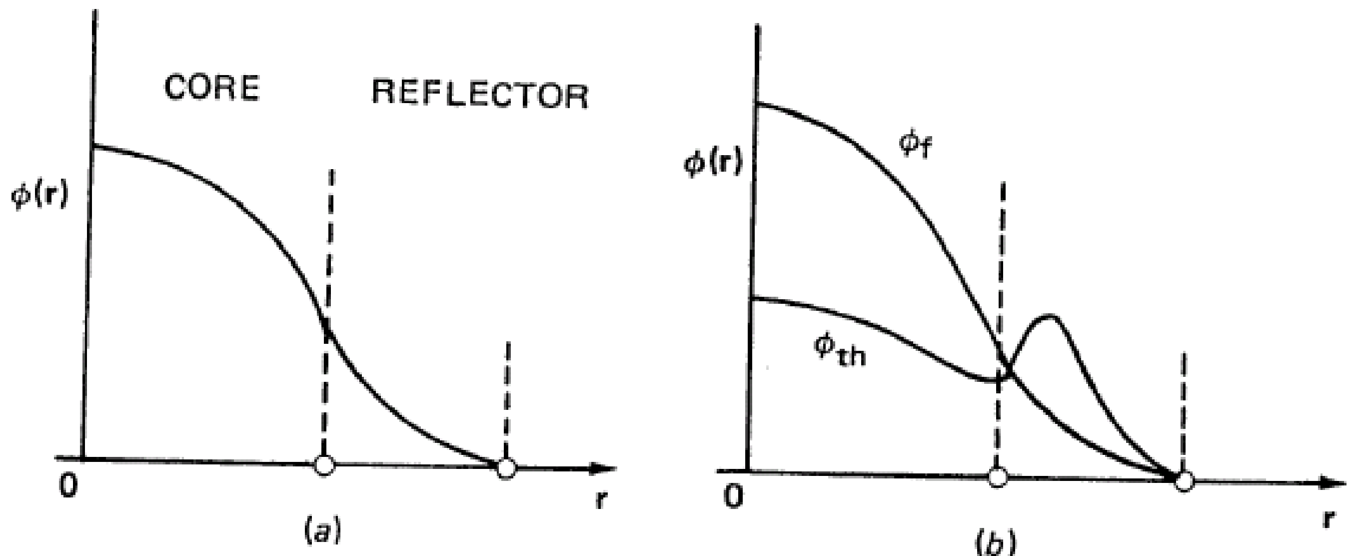


FIGURE 4-7

Comparison of (*a*) one- and (*b*) two-energy-group (fast and thermal) fluxes in core and reflector regions of a spherical reactor. (Adapted from *Reactor Analysis*, by R. V. Megrehblian and D. K. Holmes, © 1960 by McGraw-Hill Book Company, Inc. Used by permission of McGraw-Hill Book Company.)

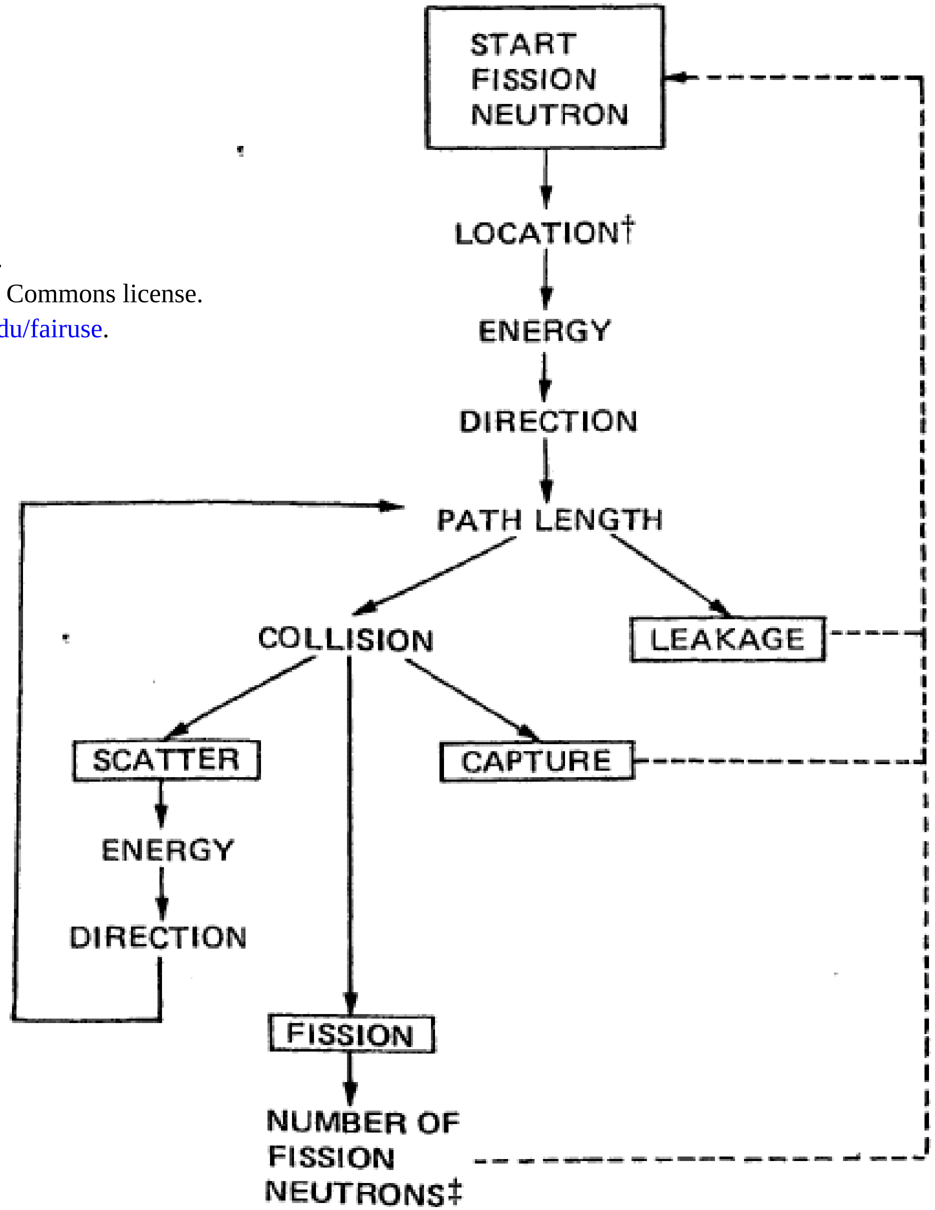


FIGURE 4-8

Flow diagram for an analog Monte Carlo method used to calculate the effective multiplication factor k. †Locations for generation N based on fission points from generation $N-1$; ‡record neutron number and fission location for generation $N+1$ starting locations.

Boltzman Equation

- Neutron Transport Theory
- Fundamental Equation of neutron interaction

Homework

- Knief Chapter 4
  - Problems: 4.3, 5, 8, 10, 11, 14, 15

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